

**Chapter**

**3**

**3**

# **Numerical Calculations**

- 3-1 Before Performing a Calculation**
- 3-2 Differential Calculations**
- 3-3 Quadratic Differential Calculations**
- 3-4 Integration Calculations**
- 3-5 Maximum/Minimum Value Calculations**
- 3-6 Summation ( $\Sigma$ ) Calculations**

## 3-1 Before Performing a Calculation



P.27

The following describes the items that are available in the menus you use when performing Solve, differential/ quadratic differential, integration, maximum/ minimum value, and  $\Sigma$  calculations.

When the option menu is on the display, press  $\boxed{F4}$  (CALC) to display the function analysis menu. The items of this menu are used when performing specific types of calculations.

- **{Solve}**/ **$\frac{d}{dx}$** / **$\frac{d^2}{dx^2}$** / **$\int dx$**  ... {solve}/**{differential}**/**{quadratic differential}**/**{integration}** calculations
- **{FMin}**/**{FMax}**/ **$\Sigma$**  ... {minimum value}/**{maximum value}**/ **$\Sigma$  (sigma)** calculations



### Solve calculations

The following is the syntax for using the Solve function in a program.

Solve(  $f(x)$ ,  $n$ ,  $a$ ,  $b$  )

Upper limit  
Lower limit  
Initial estimated value

- There are two different input methods that can be used for Solve calculations: direct assignment and variable table input.

With the direct assignment method (the one described here), you assign values directly to variables. This type of input is identical to that used with the Solve command used in the PRGM Mode.

Variable table input is used with the Solve function in the EQUA Mode. This input method is recommend for most normal Solve function input.



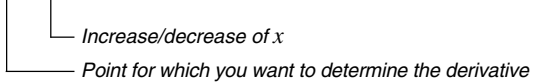
P.394

P.107

# 3-2 Differential Calculations

To perform differential calculations, first display the function analysis menu, and then input the values shown in the formula below.

$$\boxed{\text{F2}} \boxed{(d/dx)} \boxed{f(x)} \boxed{\blacktriangleright} \boxed{a} \boxed{\blacktriangleright} \boxed{\Delta x} \boxed{\text{D}}$$



$$d/dx (f(x), a, \Delta x) \Rightarrow \frac{d}{dx} f(a)$$

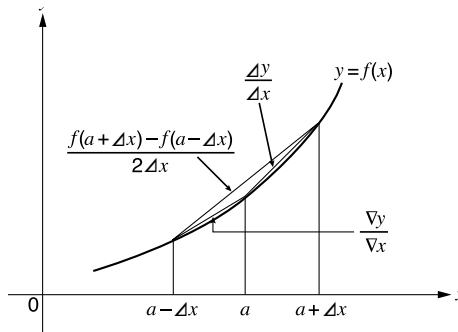
The differentiation for this type of calculation is defined as:

$$f'(a) = \lim_{\Delta x \rightarrow 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

In this definition, *infinitesimal* is replaced by a *sufficiently small*  $\Delta x$ , with the value in the neighborhood of  $f'(a)$  calculated as:

$$f'(a) \cong \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

In order to provide the best precision possible, this unit employs central difference to perform differential calculations. The following illustrates central difference.



The slopes of point  $a$  and point  $a + \Delta x$ , and of point  $a$  and point  $a - \Delta x$  in function  $y = f(x)$  are as follows:

$$\frac{f(a + \Delta x) - f(a)}{\Delta x} = \frac{\Delta y}{\Delta x}, \quad \frac{f(a) - f(a - \Delta x)}{\Delta x} = \frac{\nabla y}{\nabla x}$$

In the above,  $\Delta y/\Delta x$  is called the forward difference, while  $\nabla y/\nabla x$  is the backward difference. To calculate derivatives, the unit takes the average between the value of  $\Delta y/\Delta x$  and  $\nabla y/\nabla x$ , thereby providing higher precision for derivatives.

This average, which is called the *central difference*, is expressed as:

$$f'(a) = \frac{1}{2} \left( \frac{f(a + \Delta x) - f(a)}{\Delta x} + \frac{f(a) - f(a - \Delta x)}{\Delta x} \right)$$

$$= \frac{f(a + \Delta x) - f(a - \Delta x)}{2\Delta x}$$

**•To perform a differential calculation**

**Example** To determine the derivative at point  $x = 3$  for the function  $y = x^3 + 4x^2 + x - 6$ , when the increase/decrease of  $x$  is defined as  $\Delta x = 1E - 5$

Input the function  $f(x)$ .

**AC** **OPTN** **F4** (CALC) **F2** (d/dx) **↵** **3** **+** **4** **↵** **x<sup>2</sup>** **+** **↵** **-** **6** **↵**

Input point  $x = a$  for which you want to determine the derivative.

**3** **↵**

Input  $\Delta x$ , which is the increase/decrease of  $x$ .

**1** **EXP** **(-)** **5** **)**

**EXE**

d/dx(X^3+4X^2+X-6,3,1E-5)
52

- In the function  $f(x)$ , only X can be used as a variable in expressions. Other variables (A through Z, r,  $\theta$ ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of  $\Delta x$  and the closing parenthesis can be omitted. If you omit  $\Delta x$ , the calculator automatically uses a value for  $\Delta x$  that is appropriate for the derivative value you are trying to determine.
- Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.

## ■ Applications of Differential Calculations

- Differentials can be added, subtracted, multiplied or divided with each other.

$$\frac{d}{dx} f(a) = f'(a), \quad \frac{d}{dx} g(a) = g'(a)$$

Therefore:

$$f'(a) + g'(a), f'(a) \times g'(a), \text{ etc.}$$

- Differential results can be used in addition, subtraction, multiplication, and division, and in functions.

$$2 \times f'(a), \log(f'(a)), \text{ etc.}$$

- Functions can be used in any of the terms ( $f(x)$ ,  $a$ ,  $\Delta x$ ) of a differential.

$$\frac{d}{dx} (\sin x + \cos x, \sin 0.5), \text{ etc.}$$

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or  $\Sigma$  calculation expression inside a differential calculation term.



- Pressing **AC** during calculation of a differential (while the cursor is not shown on the display) interrupts the calculation.
- Always use radians (Rad Mode) as the angle unit when performing trigonometric differentials.

## 3-3 Quadratic Differential Calculations

[OPTN]-[CALC]-[ $d^2/dx^2$ ]

After displaying the function analysis menu, you can input quadratic differentials using either of the two following formats.

$$\boxed{F3} (d^2/dx^2) f(x) \boxed{\blacktriangleright} a \boxed{\blacktriangleright} n \boxed{\blacktriangleright}$$

Final boundary ( $n = 1$  to  $15$ )

Differential coefficient point

$$\frac{d^2}{dx^2} (f(x), a, n) \Rightarrow \frac{d^2}{dx^2} f(a)$$

Quadratic differential calculations produce an approximate differential value using the following second order differential formula, which is based on Newton's polynomial interpretation.

$$f''(x) = \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h)}{12h^2}$$

In this expression, values for "sufficiently small increments of  $x$ " are sequentially calculated using the following formula, with the value of  $m$  being substituted as  $m = 1, 2, 3$  and so on.

$$h = \frac{1}{5^m}$$

The calculation is finished when the value of  $f''(x)$  based on the value of  $h$  calculated using the last value of  $m$ , and the value of  $f''(x)$  based on the value of  $h$  calculated using the current value of  $m$  are identical before the upper  $n$  digit is reached.

- Normally, you should not input a value for  $n$ . It is recommended that you only input a value for  $n$  when required for calculation precision.
- Inputting a larger value for  $n$  does not necessarily produce greater precision.

### •To perform a quadratic differential calculation

**Example** To determine the quadratic differential coefficient at the point where  $x = 3$  for the function  $y = x^3 + 4x^2 + x - 6$   
Here we will use a final boundary value of  $n = 6$ .

Input the function  $f(x)$ .

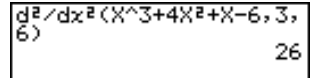
$$\boxed{AC} \boxed{OPTN} \boxed{F4} (\text{CALC}) \boxed{F3} (d^2/dx^2) \boxed{X.\theta.T} \boxed{\wedge} \boxed{3} \boxed{+}$$

$$\boxed{4} \boxed{X.\theta.T} \boxed{x^2} \boxed{+} \boxed{X.\theta.T} \boxed{-} \boxed{6} \boxed{\blacktriangleright}$$

Input 3 as point  $a$ , which is the differential coefficient point.



Input 6 as  $n$ , which is final boundary.



- In the function  $f(x)$ , only  $X$  can be used as a variable in expressions. Other variables ( $A$  through  $Z$ ,  $r$ ,  $\theta$ ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of the final boundary value  $n$  and the closing parenthesis can be omitted.
- Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.

### ■ Quadratic Differential Applications

- Arithmetic operations can be performed using two quadratic differentials.

$$\frac{d^2}{dx^2} f(a) = f''(a), \frac{d^2}{dx^2} g(a) = g''(a)$$

Therefore:

$$f''(a) + g''(a), f''(a) \times g''(a), \text{ etc.}$$

- The result of a quadratic differential calculation can be used in a subsequent arithmetic or function calculation.

$$2 \times f''(a), \log(f''(a)), \text{ etc.}$$

- Functions can be used within the terms ( $f(x)$ ,  $a$ ,  $n$ ) of a quadratic differential expression.

$$\frac{d^2}{dx^2} (\sin x + \cos x, \sin 0.5), \text{ etc.}$$

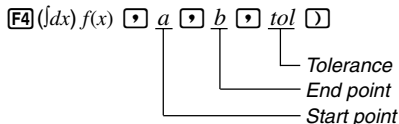
- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or  $\Sigma$  calculation expression inside of a quadratic differential calculation term.



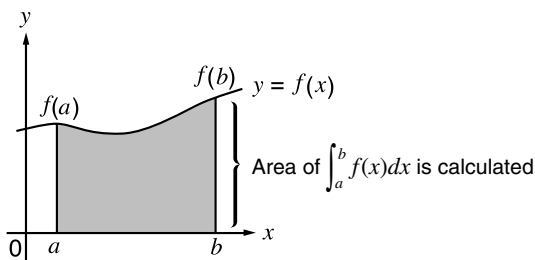
- Use only integers within the range of 1 to 15 for the value of final boundary  $n$ . Use of a value outside this range produces an error.
- You can interrupt an ongoing quadratic differential calculation by pressing the **AC** key.
- Always use radians (Rad Mode) as the angle unit when performing trigonometric quadratic differentials.

To perform integration calculations, first display the function analysis menu and then input the values in one of the formulas shown below.

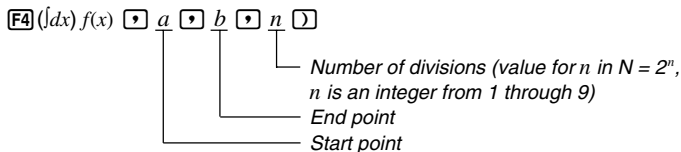
### Gauss-Kronrod Rule



$$\int(f(x), a, b, tol) \Rightarrow \int_a^b f(x)dx$$



### Simpson's Rule



$$\int(f(x), a, b, n) \Rightarrow \int_a^b f(x)dx, N = 2^n$$

As shown in the illustration above, integration calculations are performed by calculating integral values from  $a$  to  $b$  for the function  $y = f(x)$  where  $a \leq x \leq b$ , and  $f(x) \geq 0$ . This in effect calculates the surface area of the shaded area in the illustration.

\* When  $f(x) < 0$  on  $a \leq x \leq b$ , the surface area calculation produces negative values (surface area below the  $x$ -axis).

## ■ Changing Integration Calculation Methods

This calculator can use either Gauss-Kronrod Rule or Simpson's Rule to perform integration calculations. To select a method, display the set up screen and select either "**Gaus**" (for Gauss-Kronrod Rule) or "**Simp**" (for Simpson's Rule) for the Integration item.

All of the explanations in this manual use Gauss-Kronrod Rule.





## ● To perform an integration calculation

**Example** To perform the integration calculation for the function shown below, with a tolerance of “tol” =  $1E-4$

$$\int_1^5 (2x^2 + 3x + 4) dx$$

Input the function  $f(x)$ .

AC OPTN F4 (CALC) F4 (∫dx) 2 X,θT x² + 3 X,θT + 4 )

Input the start point and end point.

1 ) 5 )

Input the tolerance value.

1 EXP (-) 4 ) EXE

$\int(2X^2+3X+4, 1, 5, 1E-4)$   
134.6666667

- In the function  $f(x)$ , only X can be used as a variable in expressions. Other variables (A through Z,  $r$ ,  $\theta$ ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of “tol” in Gauss-Kronrod Rule, “n” in Simpson’s Rule, and closing parenthesis with both rules can be omitted. If you omit “tol”, the calculator automatically uses a value of  $1E-5$ . In the case of “n”, the calculator automatically selects the most appropriate value.
- Integration calculations can take a long time to complete.

## ■ Application of Integration Calculation

- Integrals can be used in addition, subtraction, multiplication or division.

$$\int_a^b f(x) dx + \int_c^d g(x) dx, \text{ etc.}$$

- Integration results can be used in addition, subtraction, multiplication or division, in functions.

$$2 \times \int_a^b f(x) dx, \text{ etc. } \log \left( \int_a^b f(x) dx \right), \text{ etc.}$$

- Functions can be used in any of the terms ( $f(x)$ ,  $a$ ,  $b$ ,  $n$ ) of an integral.

$$\int_{\sin 0.5}^{\cos 0.5} (\sin x + \cos x) dx = \int (\sin x + \cos x, \sin 0.5, \cos 0.5, 5)$$

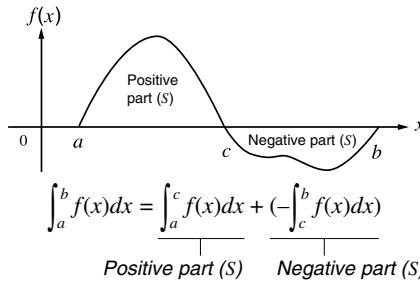
- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or  $\Sigma$  calculation expression inside of an integration calculation term.



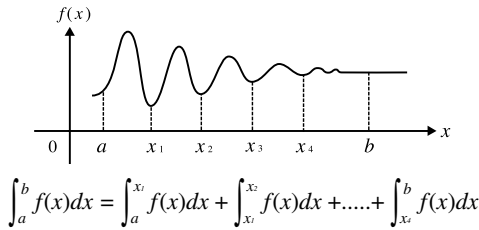
- Pressing **[AC]** during calculation of an integral (while the cursor is not shown on the display) interrupts the calculation.
- Always use radians (Rad Mode) as the angle unit when performing trigonometric integrations.
- Factors such as the type of function being used, positive and negative values within divisions, and the division where integration is being performed can cause significant error in integration values and erroneous calculation results.

Note the following points to ensure correct integration values.

- (1) When cyclical functions for integration values become positive or negative for different divisions, perform the calculation for single cycles, or divide between negative and positive, and then add the results together.



- (2) When minute fluctuations in integration divisions produce large fluctuations in integration values, calculate the integration divisions separately (divide the large fluctuation areas into smaller divisions), and then add the results together.

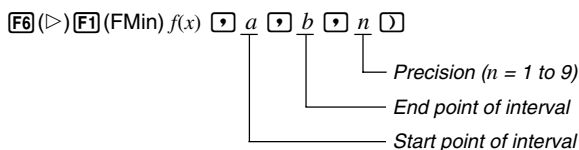


## 3-5 Maximum/Minimum Value Calculations

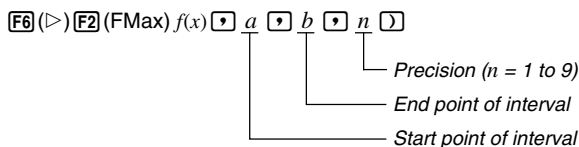
[OPTN]-[CALC]-[FMin]/[FMax]

After displaying the function analysis menu, you can input maximum/minimum calculations using the formats below, and solve for the maximum and minimum of a function within interval  $a \leq x \leq b$ .

### •Minimum Value



### •Maximum Value



### •To perform maximum/minimum value calculations

**Example 1** To determine the minimum value for the interval defined by start point  $a = 0$  and end point  $b = 3$ , with a precision of  $n = 6$  for the function  $y = x^2 - 4x + 9$

Input  $f(x)$ .

$\boxed{\text{AC}}\boxed{\text{OPTN}}\boxed{\text{F4}}(\text{CALC})\boxed{\text{F6}}(\triangleright)\boxed{\text{F1}}(\text{FMin})\boxed{\text{X,0T}}\boxed{x^2}\boxed{-}\boxed{4}\boxed{\text{X,0T}}\boxed{+}\boxed{9}\boxed{\triangleright}$

Input the interval  $a = 0$ ,  $b = 3$ .

$\boxed{0}\boxed{\triangleright}\boxed{3}\boxed{\triangleright}$

Input the precision  $n = 6$ .

$\boxed{6}\boxed{\triangleright}$

$\boxed{\text{EXE}}$

Ans
1 [ ]
2 [ ]

**Example 2** To determine the maximum value for the interval defined by start point  $a = 0$  and end point  $b = 3$ , with a precision of  $n = 6$  for the function  $y = -x^2 + 2x + 2$

Input  $f(x)$ .

**AC** **OPTN** **F4** (CALC) **F6** ( $\triangleright$ ) **F2** (FMax) **(←)** **X,θ,T** **x<sup>2</sup>** **+** **2** **X,θ,T** **+** **2** **▸**

Input the interval  $a = 0$ ,  $b = 3$ .

**0** **▸** **3** **▸**

Input the precision  $n = 6$ .

**6** **)**

**EXE**



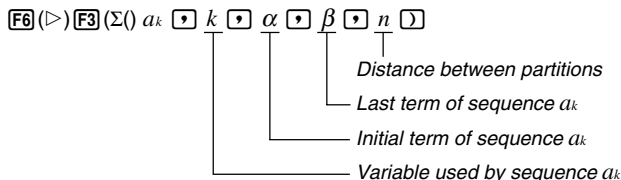
- In the function  $f(x)$ , only  $X$  can be used as a variable in expressions. Other variables ( $A$  through  $Z$ ,  $r$ ,  $\theta$ ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
  - Input of  $n$  and the closing parenthesis following the precision value can be omitted.
  - Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.
  - Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or  $\Sigma$  calculation expression inside of a maximum/minimum calculation term.
  - Inputting a larger value for  $n$  increases the precision of the calculation, but it also increases the amount of time required to perform the calculation.
- 
- The value you input for the end point of the interval ( $b$ ) must be greater than the value you input for the start point ( $a$ ). Otherwise an error is generated.
  - You can interrupt an ongoing maximum/minimum calculation by pressing the **AC** key.
  - You can input an integer in the range of 1 to 9 for the value of  $n$ . Using any value outside this range causes an error.



## 3-6 Summation ( $\Sigma$ ) Calculations

[OPTN]-[CALC]-[ $\Sigma$ ]

To perform  $\Sigma$  calculations, first display the function analysis menu, and then input the values shown in the formula below.



$$\Sigma(a_k, k, \alpha, \beta, n) \Rightarrow \sum_{k=\alpha}^{\beta} a_k$$

$\Sigma$  calculation is the calculation of the partial sum of sequence  $a_k$ , using the following formula.

$$S = a_{\alpha} + a_{\alpha+1} + \dots + a_{\beta} = \sum_{k=\alpha}^{\beta} a_k$$

### ■ Example $\Sigma$ Calculation

**Example** To calculate the following:

$$\sum_{k=2}^6 (k^2 - 3k + 5)$$

Use  $n = 1$  as the distance between partitions.

Input sequence  $a_k$ .

$\boxed{\text{AC}} \boxed{\text{OPTN}} \boxed{\text{F4}} (\text{CALC}) \boxed{\text{F6}} (\text{>}) \boxed{\text{F3}} (\Sigma) \boxed{\text{ALPHA}} \boxed{\text{K}} \boxed{\text{x}^2} \boxed{-} \boxed{3} \boxed{\text{ALPHA}} \boxed{\text{K}} \boxed{+} \boxed{5} \boxed{\text{ ] }}$

Input variable used by sequence  $a_k$ .

$\boxed{\text{ALPHA}} \boxed{\text{K}} \boxed{\text{ ] }}$

Input the initial term of sequence  $a_k$  and last term of sequence  $a_k$ .

$\boxed{2} \boxed{\text{ ] }} \boxed{6} \boxed{\text{ ] }}$

Input  $n$ .

$\boxed{1} \boxed{\text{ ] }}$

$\boxed{\text{EXE}}$

$\Sigma(K^2-3K+5, K, 2, 6, 1)$  55

- You can use only one variable in the function for input sequence  $a_k$ .
- Input integers only for the initial term of sequence  $a_k$  and last term of sequence  $a_k$ .
- Input of  $n$  and the closing parentheses can be omitted. If you omit  $n$ , the calculator automatically uses  $n = 1$ .

#### ■ $\Sigma$ Calculation Applications

- Arithmetic operations using  $\Sigma$  calculation expressions

Expressions: 
$$S_n = \sum_{k=1}^n a_k, T_n = \sum_{k=1}^n b_k$$

Possible operations:  $S_n + T_n, S_n - T_n$ , etc.

- Arithmetic and function operations using  $\Sigma$  calculation results

$2 \times S_n, \log(S_n)$ , etc.

- Function operations using  $\Sigma$  calculation terms ( $a_k, k$ )

$\Sigma(\sin k, k, 1, 5)$ , etc.

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or  $\Sigma$  calculation expression inside of a  $\Sigma$  calculation term.



- Make sure that the value used as the final term  $\beta$  is greater than the value used as the initial term  $\alpha$ . Otherwise, an error will occur.
- To interrupt an ongoing  $\Sigma$  calculation (indicated when the cursor is not on the display), press the  $\overline{\text{AC}}$  key.