Chapter 6

Matrix Calculations

26 matrix memories (Mat A through Mat Z) plus a Matrix Answer Memory (MatAns), make it possible to perform the following matrix operations.

- Addition, subtraction, multiplication
- Scalar multiplication calculations
- Determinant calculations
- Matrix transposition
- Matrix inversion
- Matrix squaring
- Raising a matrix to a specific power
- Absolute value, integer part extraction, fractional part extraction, maximum integer calculations
- Matrix modification using matrix commands

6-1 Before Performing Matrix Calculations

In the Main Menu, select the **MAT** icon to enter the Matrix Mode and display its initial screen.

2 (row) × 2 (column) matrix

Not dimension preset

- {**DEL**}/{**DEL·A**} ... deletes {a specific matrix}/{all matrices}

- The maximum number of rows that can be specified for a matrix is 255, and the maximum number of columns is 255.

■ About Matrix Answer Memory (MatAns)

The calculator automatically stores matrix calculation results in Matrix Answer Memory. Note the following points about Matrix Answer Memory.

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- Whenever you perform a matrix calculation, the current Matrix Answer Memory contents are replaced by the new result. The previous contents are deleted and cannot be recovered.
- Inputting values into a matrix does not affect Matrix Answer Memory contents.

■ Creating a Matrix

To create a matrix, you must first define its dimensions (size) in the MATRIX list. Then you can input values into the matrix.

●To specify the dimensions of a matrix

Example **To create a 2-row × 3-column matrix in the area named Mat B**

Highlight Mat B.

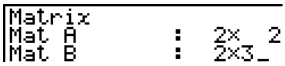
Specify the number of rows.

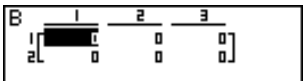
2 EXE

Specify the number of columns.

3

EXE





- All of the cells of a new matrix contain the value 0.
- If "Mem ERROR" remains next to the matrix area name after you input the dimensions, it means there is not enough free memory to create the matrix you want.

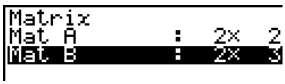
●To input cell values

Example **To input the following data into Matrix B :**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Select Mat B.

▼



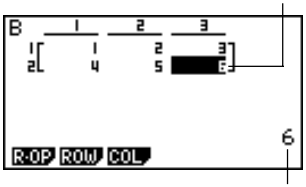
Highlighted cell (up to six digits can be displayed)

EXE

1 EXE 2 EXE 3 EXE

4 EXE 5 EXE 6 EXE

(Data is input into the highlighted cell. Each time you press EXE, the highlighting move to the next cell to the right.)



Value in currently highlighted cell

- Displayed cell values show positive integers up to six digits, and negative integers up to five digits (one digit used for the negative sign). Exponential values are shown with up to two digits for the exponent. Fractional values are not displayed.
- You can see the entire value assigned to a cell by using the cursor keys to move the highlighting to the cell whose value you want to view.
- The amount of memory required for a matrix is ten bytes per cell. This means that a 3×3 matrix requires 90 bytes of memory ($3 \times 3 \times 10 = 90$).

■ Deleting Matrices

You can delete either a specific matrix or all matrices in memory.

●To delete a specific matrix

1. While the MATRIX list is on the display, use ▲ and ▼ to highlight the matrix you want to delete.
2. Press [F1] (DEL).
3. Press [F1] (YES) to delete the matrix or [F6] (NO) to abort the operation without deleting anything.

   • The indicator “None” replaces the dimensions of the matrix you delete.

●To delete all matrices

1. While the MATRIX list is on the display, press [F2] (DEL·A).
2. Press [F1] (YES) to delete all matrices in memory or [F6] (NO) to abort the operation without deleting anything.

   • The indicator “None” is shown for all the matrices.

6-2 Matrix Cell Operations

Use the following procedure to prepare a matrix for cell operations.

1. While the MATRIX list is on the display, use ▲ and ▼ to highlight the name of the matrix you want to use.
2. Press [EXE] and the function menu with the following items appears.
   - {**R·OP**} ... {row calculation menu}
   - {**ROW**}/{**COL**} ... {row}/{column} operation menu

All of the following examples use Matrix A recalled by the above operation.

■ Row Calculations

The following menu appears whenever you press [F1] (R·OP) while a recalled matrix is on the display.

- {**Swap**} ... {row swap}
- {×**Rw**} ... {product of specified row and scalar}
- {×**Rw+**} ... {addition of one row and the product of a specified row with a scalar}
- {**Rw+**} ... {addition of specified row to another row}

●To swap two rows

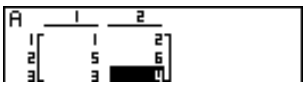
Example **To swap rows 2 and 3 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F1](R·OP)[F1](Swap)

Input the number of the rows you want to swap.

[2][EXE][3][EXE]



●To calculate the product of a row

Example **To calculate the product of row 2 of the following matrix and the scalar 4 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

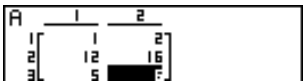
F1 (R·OP) F2 (×Rw)

Input multiplier value.

4 EXE

Specify row number.

2 EXE



●To calculate the product of a row and add the result to another row

Example **To calculate the product of row 2 of the following matrix and the scalar 4, then add the result to row 3 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F1 (R·OP) F3 (×Rw+)

Input multiplier value.

4 EXE

Specify the row number whose product should be calculated.

2 EXE

Specify the row number where result should be added.

3 EXE

●To add two rows together

Example **To add row 2 to row 3 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

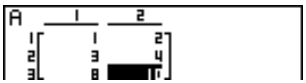
F1 (R·OP) F4 (Rw+)

Specify the row number to be added.

2 EXE

Specify the row number to be added to.

3 EXE



■ Row Operations

The following menu appears whenever you press F2 (ROW) while a recalled matrix is on the display.

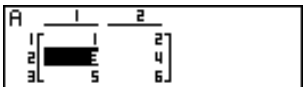
- {**DEL**} ... {delete row}
- {**INS**} ... {insert row}
- {**ADD**} ... {add row}

●To delete a row

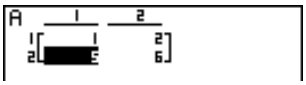
Example **To delete row 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F2 (ROW) ▼



F1 (DEL)

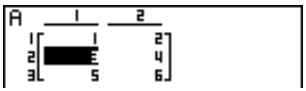


●To insert a row

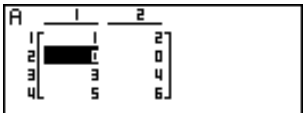
Example **To insert a new row between rows 1 and 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F2 (ROW) ▼



F2 (INS)

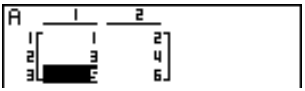


●To add a row

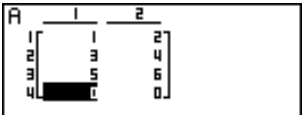
Example **To add a new row below row 3 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F2](ROW)▼▼



[F3](ADD)



■ Column Operations

The following menu appears whenever you press [F3] (COL) while a recalled matrix is on the display.

- {**DEL**} ... {delete column}
- {**INS**} ... {insert column}
- {**ADD**} ... {add column}

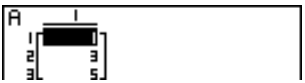
●To delete a column

Example **To delete column 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F3](COL)▶

[F1](DEL)



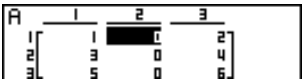
●To insert a column

Example **To insert a new column between columns 1 and 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F3(COL)▶

F2(INS)



●To add a column

Example **To add a new column to the right of column 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F3(COL)▶

F3(ADD)

6-3 Modifying Matrices Using Matrix Commands

[OPTN]-[MAT]

●To display the matrix commands

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1. From the Main Menu, select the **RUN** icon and press [EXE].
2. Press [OPTN] to display the option menu.
3. Press [F2] (MAT) to display the matrix operation menu.

The following describes only the matrix command menu items that are used for creating matrices and inputting matrix data.

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- {**Mat**} ... {Mat command (matrix specification)}
- {**M→L**} ... {Mat→List command (assign contents of selected column to list file)}
- {**Aug**} ... {Augment command (link two matrices)}
- {**Iden**} ... {Identity command (identity matrix input)}
- {**Dim**} ... {Dim command (dimension check)}
- {**Fill**} ... {Fill command (identical cell values)}

■ Matrix Data Input Format

The following shows the format you should use when inputting data to create a matrix using the matrix operation menu's Mat command.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

= [[a_{11}, a_{12}, ..., a_{1n}] [a_{21}, a_{22}, ..., a_{2n}] .... [a_{m1}, a_{m2}, ..., a_{mn}]]
→ Mat [letter A through Z]

- The maximum value of both m and n is 255.

Example 1 **To input the following data as Matrix A :**

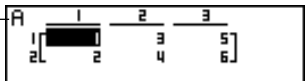
$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

[OPTN] [F2] (MAT)
[SHIFT] [(] [SHIFT] [(] [1] [,] [3] [,] [5]
[SHIFT] [)] [SHIFT] [(] [2] [,] [4] [,] [6]
[SHIFT] [)] [SHIFT] [)] [→] [F1] (Mat) [ALPHA] [A]

```
[[1,3,5][2,4,6]]→Mat
A_
```

[EXE]

Matrix name



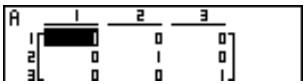
- An error occurs if memory becomes full as you are inputting data.
- You can also use the above format inside a program that inputs matrix data.

●To input an identity matrix

Use the matrix operation menu's Identity command ([F1]) to create an identity matrix.

Example 2 **To create a 3 × 3 identity matrix as Matrix A**

[OPTN] [F2] (MAT) [F6] (▷) [F1] (Iden)
[3] [→] [F6] (▷) [F1] (Mat) [ALPHA] [A] [EXE]
Number of rows/columns



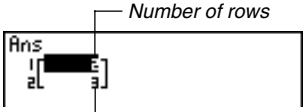
●To check the dimensions of a matrix

Use the matrix operation menu's Dim command ([F2]) to check the dimensions of an existing matrix.

Example 3 **To check the dimensions of Matrix A, which was input in Example 1**

[OPTN] [F2] (MAT) [F6] (▷) [F2] (Dim) [F6] (▷)
[F1] (Mat) [ALPHA] [A] [EXE]

Number of rows



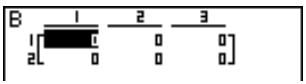
Number of columns

The display shows that Matrix A consists of two rows and three columns.

You can also use {Dim} to specify the dimensions of the matrix.

Example 4 **To specify dimensions of 2 rows and 3 columns for Matrix B**

[SHIFT] [{] [2] [,] [3] [SHIFT] [}] [→] [OPTN]
[F2] (MAT) [F6] (▷) [F2] (Dim) [F6] (▷)
[F1] (Mat) [ALPHA] [B] [EXE]



■ Modifying Matrices Using Matrix Commands

You can also use matrix commands to assign values to and recall values from an existing matrix, to fill in all cells of an existing matrix with the same value, to combine two matrices into a single matrix, and to assign the contents of a matrix column to a list file.

●To assign values to and recall values from an existing matrix

Use the following format with the matrix operation menu's Mat command ([F1]) to specify a cell for value assignment and recall.

Mat X [*m*, *n*]

X ..... matrix name (A through Z, or Ans)

m ..... row number

n ..... column number

Example 1 **Assign 10 to the cell at row 1, column 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[1] [0] [→] [OPTN] [F2] (MAT) [F1] (Mat)
[ALPHA] [A] [SHIFT] [(] [1] [,] [2] [SHIFT] [)] [EXE]

```
10→Mat A[1,2]
                10
```

Example 2 **Multiply the value in the cell at row 2, column 2 of the above matrix by 5**

[OPTN] [F2] (MAT) [F1] (Mat)
[ALPHA] [A] [SHIFT] [(] [2] [,] [2] [SHIFT] [)]
[×] [5] [EXE]

```
Mat A[2,2]×5
                20
```

●To fill a matrix with identical values and to combine two matrices into a single matrix

Use the matrix operation menu's Fill command ([F3]) to fill all the cells of an existing matrix with an identical value, or the Augment command ([F5]) to combine two existing matrices into a single matrix.

Example 1 **To fill all of the cells of Matrix A with the value 3**

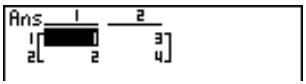
[OPTN] [F2] (MAT) [F6] (▷) [F3] (Fill)
[3] [,] [F6] (▷) [F1] (Mat) [ALPHA] [A] [EXE]
Filler value

```
Fill(3,Mat A
                Done
```

Example 2 **To combine the following two matrices :**

$$\mathbf{A} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

OPTN F2 (MAT) F5 (Aug) F1 (Mat)
ALPHA A , F1 (Mat) ALPHA B EXE



- The two matrices you combine must have the same number of rows. An error occurs if you try to combine two matrices that have different numbers of rows.

●To assign the contents of a matrix column to a list file

Use the following format with the matrix operation menu's Mat→List command (F2) to specify a column and a list file.

Mat → List (Mat X, m) → List n

X = matrix name (A through Z, or Ans)

m = column number

n = list number

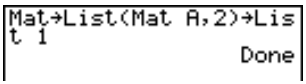
Example **To assign the contents of column 2 of the following matrix to list file 1 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

OPTN F2 (MAT) F2 (M→L) F1 (Mat)
ALPHA A , 2) →

Column number

OPTN F1 (LIST) F1 (List) 1 EXE



You can use Matrix Answer Memory to assign the results of the above matrix input and edit operations to a matrix variable. To do so, use the following syntax.

- Fill (n, Mat α) → Mat β
- Augment (Mat α, Mat β) → Mat γ

In the above, α, β, and γ are any variable names A through Z, and n is any value.

The above does not affect the contents of Matrix Answer Memory.

6-4 Matrix Calculations

[OPTN]-[MAT]

Use the matrix command menu to perform matrix calculation operations.

●To display the matrix commands

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1. From the Main Menu, select the **RUN** icon and press EXE.
2. Press OPTN to display the option menu.
3. Press F2 (MAT) to display the matrix command menu.

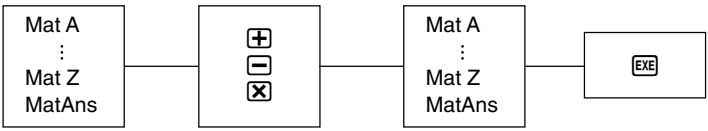
The following describes only the matrix commands that are used for matrix arithmetic operations.

- {**Mat**} ... {Mat command (matrix specification)}
- {**Det**} ... {Det command (determinant command)}
- {**Trn**} ... {Trn command (transpose matrix command)}
- {**Iden**} ... {Identity command (identity matrix input)}

All of the following examples assume that matrix data is already stored in memory.

■ Matrix Arithmetic Operations

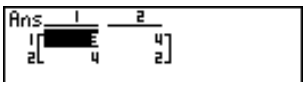
Matrix 1 **Arithmetic operator key** **Matrix 2**



Example 1 **To add the following two matrices (Matrix A + Matrix B) :**

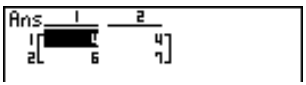
$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

F1 (Mat) ALPHA A +
F1 (Mat) ALPHA B EXE



Example 2 **To multiply the two matrices in Example 1 (Matrix A × Matrix B)**

F1 (Mat) ALPHA A ×
F1 (Mat) ALPHA B EXE

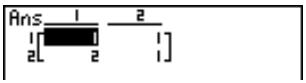


- The two matrices must have the same dimensions in order to be added or subtracted. An error occurs if you try to add or subtract matrices of different dimensions.
- For multiplication, the number of columns in Matrix 1 must match the number of rows in Matrix 2. Otherwise, an error occurs.

- You can use an identity matrix in place of Matrix 1 or Matrix 2 in the matrix arithmetic format. Use the matrix command menu's Identity command ([F1]) to input the identity matrix.

Example 3 **To multiply Matrix A (from Example 1) by a 2 × 2 identity matrix**

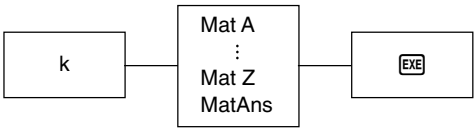
[F1](Mat)[ALPHA][A][×]
[F6](▷)[F1](Iden)[2][EXE]
Number of rows and columns.



■ Matrix Scalar Multiplication

The following is the format for calculating a matrix scalar multiplication, which multiplies the value in each cell of the matrix by the same value.

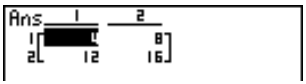
Scalar value **Matrix**



Example **Calculate the product of the following matrix using a multiplier value of 4 :**

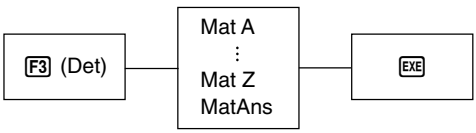
Matrix A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

[4][F1](Mat)[ALPHA][A][EXE]



■ Determinant

Matrix



Example **Obtain the determinant for the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & -2 & 0 \end{bmatrix}$$

[F3](Det)[F1](Mat)[ALPHA][A][EXE]

- Determinants can be obtained only for square matrices (same number of rows and columns). Trying to obtain a determinant for a matrix that is not square produces an error.

- The determinant of a 2×2 matrix is calculated as shown below.

$$|A| = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

- The determinant of a 3×3 matrix is calculated as shown below.

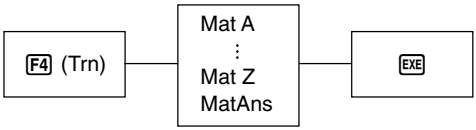
$$|A| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

■ Matrix Transposition

A matrix is transposed when its rows become columns and its columns become rows. The following is the format for matrix transposition.

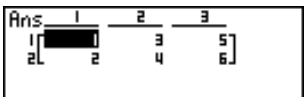
Matrix



Example **To transpose the following matrix :**

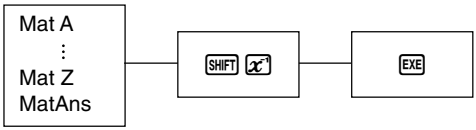
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F4](Trn)[F1](Mat)[ALPHA][A][EXE]



■ Matrix Inversion

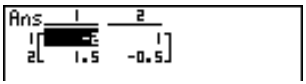
Matrix



Example **To invert the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A SHIFT x^{-1} EXE



- Only square matrices (same number of rows and columns) can be inverted. Trying to invert a matrix that is not square produces an error.
- A matrix with a value of zero cannot be inverted. Trying to invert a matrix with value of zero produces an error.
- Calculation precision is affected for matrices whose value is near zero.

- A matrix being inverted must satisfy the conditions shown below.

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}^{-1}\,\mathbf{A} = \mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The following shows the formula used to invert Matrix A into inverse matrix A^{-1}.

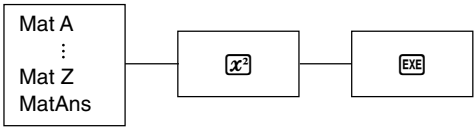
$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Note that $ad - bc \neq 0$.

■ Squaring a Matrix

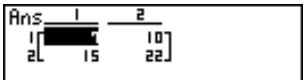
Matrix



Example **To square the following matrix :**

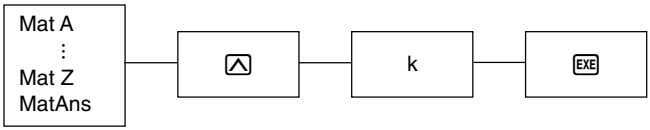
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A x² EXE



■ Raising a Matrix to a Power

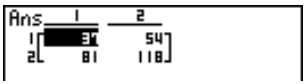
Matrix **Natural number**



Example **To raise the following matrix to the third power :**

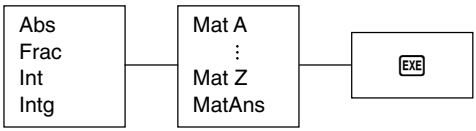
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A ^ 3 EXE



■ Determining the Absolute Value, Integer Part, Fraction Part, and Maximum Integer of a Matrix

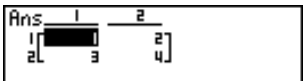
Function command **Matrix**



Example **To determine the absolute value of the following matrix :**

Matrix A = $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

OPTN F6 (▷) F4 (NUM) F1 (Abs)
OPTN F2 (MAT) F1 (Mat) ALPHA A EXE



- Determinants and inverse matrices are calculated using the elimination method, so errors (such as dropped digits) may be generated.
- Matrix operations are performed individually on each cell, so calculations may require considerable time to complete.
- The calculation precision of displayed results for matrix calculations is ± 1 at the least significant digit.
- If a matrix calculation result is too large to fit into Matrix Answer Memory, an error occurs.
- You can use the following operation to transfer Matrix Answer Memory contents to another matrix (or when Matrix Answer Memory contains a determinant to a variable).

  MatAns → Mat α

  In the above, α is any variable name A through Z. The above does not affect the contents of Matrix Answer Memory.