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MULTI-OBJECTIVE PORTFOLIO OPTIMISATION OF UPSTREAM PETROLEUM PROJECTS

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ABSTRACT

The shareholders of E&P companies evaluate the future performance of these companies in terms of multiple performance attributes. Hence, E&P decision makers have the task of allocating limited resources to available project proposals to deliver the best performance on these various attributes. Additionally, the performance of these proposals on these attributes is uncertain and the attributes of the various proposals are usually correlated. As a result of the above, the E&P portfolio optimisation decision setting is characterised by multiple attributes with uncertain future performance.

Most recent contributions in the E&P portfolio optimisation arena seek to adapt modern financial portfolio theory concepts to the E&P project portfolio selection problem. These contributions generally focus on understanding the tradeoffs between risk and return for the attribute NPV while acknowledging the presence of correlation among the assets of the portfolio. The result is usually an efficient frontier where one objective is set over the expected value of the NPV and the other is set over a risk metric calculated from the same attribute where, typically, the risk metric has a closed form solution (e.g., variance, standard deviation, semi-standard deviation). However, this methodology fails to acknowledge the presence of multiple attributes in the E&P decision setting.

To fill this gap, this thesis proposes a decision support model to optimise risk and return objectives extracted from the NPV attribute and from other financial and/or operational attributes simultaneously. The result of this approach is an approximate Pareto front that explicitly shows the tradeoffs among these objectives whilst honouring intra-project and inter-project correlations. Intra-project correlations are incorporated into the optimisation by integrating the single project models to the portfolio model to be optimised. Inter-project correlation is included by modelling of the oil price a global variable. Additionally, the model uses a multi-objective simulation-optimisation approach and hence it overcomes the need of using risk metrics with closed form solutions.

The model is applied to a set of realistic hypothetical offshore E&P projects. The results show the presence of complex relationships among the objectives in the approximate Pareto set. The ability of the method to unveil these relationships hopes to bring more insight to the decision makers and hence promote better investment decisions in the E&P industry.

STATEMENT OF ORIGINALITY

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

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LIST OF SELECTED ACRONYMS AND ABBREVIATIONS

Abex	abandonment costs
ATNCF	after tax net cash flow
Bbl	barrels
Capex	capital expenditures
DM	decision maker
E&P	downstream oil and gas exploration and production companies
E(X)	expected value of the attribute X
EconLimit	economic limit of the field
EUR	expected ultimate recovery
FixOpex	fixed operating expenditures
IntanCapex	intangible capital expenditures
M	thousand
Maxcap	maximum capacity of the production hub
MM	million
NCF	net cash flow
NPV	net present value
OOIP	original oil in place
Opex	operating expenditures
P(X>0)	probability that the attribute X will return a positive value
P10(X)	10^{th} percentile of the attribute X
P50(X)	50^{th} percentile of the attribute X
P90(X)	90^{th} percentile of the attribute X
PDF	probability density function
PSC	production sharing contract
q_t	production of the field in time t
R	reserves
ROCE	return over capital employed
SD(X)	standard deviation of the attribute X
TanCapex	tangible capital expenditure
US$	United States Dollars
VarOpex	variable operating expenditures

1 INTRODUCTION

1.1 Background

Upstream oil and gas companies continuously face the crucial decision of allocating resources to project portfolios. This decision involves multiple objectives over various economic and operational performance attributes. Individual project proposals are usually characterised in terms of these various attributes and the portfolio performance is an aggregate of the performance of the individual projects. Additionally, the future performance of project proposals in these various attributes is highly uncertain. Typically, only a subset of the project proposals can be funded with the available resources, and no feasible portfolio optimises all the relevant attributes simultaneously. This decision setting is here referred to as multiple objective portfolio optimisation under uncertainty.

However, common practice in the upstream oil and gas industry to select project portfolios fails to address the presence of multiple objectives and uncertainty in such a decision setting. Hence, this decision setting is most commonly solved with tools that optimise a single objective under the assumption of a future certain performance. Most of the upstream oil and gas portfolio optimisation literature (Orman and Duggan (1998); Ball and Savage (1999a); Ball and Savage (1999b); Brashear et al. (2000); Bratvold et al. (2003)) pinpoint the inability of conventional capital-allocation methods to adequately address the uncertain quality of this decision setting. The work of these authors mostly promotes the application of financial portfolio theory to real E&P assets. These authors suggest that the main purpose of portfolio optimisation is not to provide an individual "best" solution to the decision maker (DM) but to provide insight about the future performance of various investment strategies before the actual decision is made. Financial portfolio theory delivers this insight through the generation of a set of non-dominated[1] solutions that unveils the tradeoffs between risk and return for one performance attribute (NPV) of a financial assets portfolio.

From a seminal paper by Hightower and David (1991), many advances have been made to adapt modern portfolio theory to the E&P business. Most of these works promote the value of understanding the tradeoffs between risk and return for the NPV attribute in a project portfolio optimisation context. They propose that corporate DMs should explicitly generate an efficient set of non-dominated portfolios in terms of the expected value of the NPV and some form of risk measurement of this attribute. DMs should then use this efficient set of "optimal trade-off" to gain insight about the potential of their current project proposals. Once the DMs have studied this efficient set, they may decide to invest in one of the non-dominated portfolios in this set or look for new opportunities to include in the optimisation exercise

[1] The concept of dominance is defined in section 4.4.1.2

and then repeat the whole process until a portfolio that suits the corporate system of preferences is found.

In contrast, the upstream oil and gas literature has paid less attention to the multi-objective quality of the project portfolio selection decision setting. Walls (1995); DuBois and Howell (2000); Howell and Tyler (2001) and Simpson (2002) not only address the fact that the future performance of E&P project portfolios is uncertain but also highlight the fact that portfolio optimisation methods should account for the performance of investment portfolios in terms of other operational (e.g., production, reserves) and economical (e.g., ROCE, cash flow) attributes. However, most of these works account for these attributes either with the use of constraints (DuBois and Howell (2000); Howell and Tyler (2001)) or with the use of a multi attribute utility function (Walls (1995)) and hence do not explicitly show the tradeoffs among these attributes for the potential investment portfolios. Moreover, these works overlook the fact that, as the NPV metric, the performance of these attributes is also uncertain and hence, corporate DMs might also desire to optimise the performance of a portfolio in terms of various statistics (i.e., expected values, standard deviation, and percentiles) of other performance attributes.

To promote transparency and hence deliver more insight to corporate DMs it is instructive to extend the idea of generating a non-dominated set of portfolios in terms of risk and return statistics from one to multiple performance attributes. With this approach, both the uncertain and multi-objective qualities of the oil and gas portfolio selection problem would be honoured. In this manner E&P DMs would be able to explicitly understand the tradeoffs among risk and return statistics for various operational and economical performance attributes simultaneously and, hopefully, make better investment decisions.

1.2 Objectives and scope

In this thesis, a multi-objective E&P project portfolio selection under uncertainty problem is considered. In this decision setting, the DM or group of DMs chooses the optimal working interest for a set of project proposals subject to a budgetary constraint. The problem includes inter-project and intra-project correlations. It is assumed that the DM(s) prefers to articulate her preferences once a set of non-dominated solutions has been generated in terms of various statistics from several performance attributes. Hence no single “overall” value attribute is considered.

The objectives of this thesis are:

- To develop a decision support model able to generate a non-dominated solution set of portfolios in terms of various statistics for multiple performance attributes whilst accounting for inter and intra-project dependencies.
- To implement the model to a set of hypothetical yet realistic E&P projects and analyse the outputs and discuss the advantages and disadvantages of the decision support.

The contribution of this research attempts to be incremental to the knowledge that already exists in the literature. The main feature of the model presented in this research is that it explicitly shows the tradeoffs among different objectives to the DM. As a consequence, the preferences of the DM are needed after the optimisation problem has been solved. This contrasts with current state of the art in the literature that requires an a priori statement of the DM preferences either with the use of target levels (Howell and Tyler (2001)) or with the use of an elicited multi-attribute utility function (Walls (1995))[2].

1.3 Assumptions and limitations

In order to limit the scope of the research, it is necessary to make several assumptions. The assumptions made for this research can be classified as flexible assumptions and rigid assumptions. Flexible assumptions are related to the projects being considered for the portfolio. These assumptions can be manipulated and are made to show a practical application of the model. Rigid assumptions cannot be changed and actually show the limitations of the portfolio optimisation model presented in this work.

1.3.1.1 Flexible assumptions

- *Price dependency*. The only source of correlation between projects will be the oil price. This will be done treating the price variable as a global variable that is shared by all the projects involved.
- *Development projects*. For simplicity, the projects considered for optimisation will be projects “approved for development”. Hence the “dry hole” risk is considered to be equal to zero.
- *Three statistics*. The model uses 3 possible types of objectives per attribute:
    - Expected value.
    - Percentiles.
    - Probability of achieving a certain target level.
- *Simple reservoirs*. The production of the projects will be modelled with simple “tank models”. All the projects will be offshore oil producers.
- *Small portfolios*. This research will consider portfolios with five project proposals. This not only helps to reduce computing time but also facilitates tracing the impact of each one of the projects in the resulting portfolio.

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[2] Ehrgott, M., K. Klamroth and C. Schwehm (2004). "A MCDM approach to portfolio optimization." European Journal of operational research **25**(2): 752-770. proposes a very similar approach for stock portfolios.

- *Concessionary regime*. All the performance attributes used to characterise a project (e.g., NPV, cash flow, production) will be calculated assuming that projects are developed in countries with simple concessionary[3] fiscal terms.

1.3.1.2 Rigid assumptions

- *Optimise working interest.* This research will define a portfolio in terms of a participation vector. The working interest must be a continuous variable[4]. The model does not account for "time to invest" as a decision variable.
- *Linearly constrained optimisation*. The constraints are limited to be linear, that is, a linear combination of the decision variables.

1.4 Outline of the thesis

Chapter 2 describes the main features of the general oil and gas portfolio optimisation problem. Chapter 3 describes the basics of stochastic modelling and valuation of upstream petroleum projects using Monte Carlo simulation. The calculation of typical operational and economical attributes is briefly described. The relevance of correlations in Monte Carlo simulation is highlighted and basic concepts of utility theory are reviewed.

Chapter 4 draws on the operations research literature to provide a review of single and multi-objective optimisation concepts. The ε-constraint and value function methods are described to set a theoretical ground for discussions on chapter 5. The simulation-optimisation concept is described and the multi-objective genetic algorithm with linear constraints (MOGOL) (Medaglia (2003)) is presented and its advantages and limitations are discussed.

Chapter 5 draws on the oil and gas decision and risk analysis literature to highlight the gaps on current methods for upstream petroleum portfolio optimisation. The classic capital rationing approach, the mean-variance approach and more novel approaches as the methods proposed by Walls (1995); Howell and Tyler (2001) and Rodriguez and Galvao (2005) are discussed using the theoretical ground set in chapter 4.

Chapter 6 describes the multi-objective portfolio optimisation model proposed in this research. It explains why was the MOGOL algorithm selected to perform as a search engine. The proposed model is presented making emphasis on how to make it account for inter and intra-project dependencies through its integration with project characterisation and a mean-reverting oil price model.

[3] In most countries the government owns all mineral resources. Under concessionary regimes, the government transfers the title of the minerals to company if they are produced. The company is then subject to payment of royalties and taxes.
[4] In practice, it is rare that oil companies have complete freedom to choose the level of the working interest they would prefer to have. Hence, the working interest variable might not be continuous..

In Chapter 7 the proposed multi-objective portfolio model is applied to a set of representative projects and the results of the application are analysed.

The last chapter, Chapter 8, shows the conclusions that can be drawn from the research. The advantages and disadvantages of the proposed model are discussed and its applicability is addressed. The chapter ends stating possible areas of future research within and without the limits of the proposed model.

2 OVERVIEW OF THE GENERAL UPSTREAM OIL AND GAS PORTFOLIO OPTIMISATION PROBLEM

2.1 Introduction

The purpose of this chapter is to describe the main characteristics of the upstream oil and gas portfolio selection problem. The multi-objective and uncertain qualities of the problem are described and the impact of these characteristics on the selection of an optimisation algorithm is discussed.

2.2 The source of the problem: the budgetary constraint

Typically downstream oil & gas companies have several proposed competing projects with different scale, benefits and resources requirements. According to finance theory, in order to maximize shareholder value, the corporation should fund every available project with positive NPV (Brealey and Myers (2000)). This premise is supported by the assumption that it is possible for the corporation to borrow unlimited funds at a given interest rate.

However, in practice the assumption that an unlimited supply of capital is available for the corporation does not hold (Luenberger (1998)). Banks may impose limited credit lines or, in large organizations, investment decisions may be decentralized and limited budgets may be assigned to individual organizational units.

Moreover, other non-capital constraints may prevent E&P companies to invest in every single project with a positive NPV. Some of these scarce non-capital resources may be, for example, human resources with the necessary expertise or the availability of drilling rigs in a given development area.

In summary, it is not uncommon that even if all available projects offer attractive benefits, it may not be possible to fund them all as a result of some capital and/or non-capital resource restriction. The presence of this constraint is the origin of the **capital budgeting** problem, most known in the oil and gas literature as **project portfolio optimisation**.

In a broad sense, the portfolio optimisation problem consists in finding an optimal combination of **working interest** and a **time to invest** for each one of the project proposals that **maximises shareholder value** for a given planning horizon whilst accounting for a **resource constraint**.

2.3 Shareholder value maximisation: a single objective?

The optimisation problem previously defined is based on the fact that most E&P public companies have shareholder value maximization as their ultimate objective (DuBois and Howell (2000))[5]. As stated in the previous section, finance theory states that to maximise shareholder value it is only necessary to maximise the NPV of the investment portfolio of the company.

However, Pande (2003) shows that shareholder perception of the value of E&P firms is not only driven by NPV but also by other performance attributes like cash flow, reserves, production, ROCE and Opex. As a result, Walls (1995); DuBois and Howell (2000); Howell and Tyler (2001) indicate that oil and gas portfolio optimisation tools should account for the performance of several economical and operational attributes.

A clear empirical evidence of the relevance of the previous statement occurred in January of 2004 when giant oil group Royal Dutch Shell share price drop at least 7% in the stock markets of London and Amsterdam after announcing that 20% of their proved reserves were reclassified. The following quotes appeared in BBCNews (2004)[6]:

"Giant oil group Royal Dutch Shell has said it is trimming its figures for proved oil and gas reserves by 20%."

"Shell said it does not expect the reassessment to have any impact on its financial results, as 90% of the reserves involved remain undeveloped."

"Stunned investors promptly began a sell-off that knocked more than 7% off the Anglo-Dutch firm's share price in both London and Amsterdam."

It is possible to infer from the previous case that in order to maximise shareholder value, corporate DMs must choose a portfolio of projects that they consider delivers the best possible performance in a set of multiple economic and operational attributes. Figure 1-1 shows how the ultimate corporate objective "maximise shareholder value" has to be accomplished through the sub-objective "select optimal

[5] National oil companies pursue the objective of maximizing the value of the people of the country, which is an equivalent problem.
[6] http://news.bbc.co.uk/1/hi/business/3382045.stm

portfolio" and that this sub-objective depends on multiple mean-objectives traced on various performance attributes (i.e., maximise reserves, maximise NPV).

NOTE: This figure is included on page 7 of the print copy of the thesis held in the University of Adelaide Library.

Figure 2-1 Relationship between corporate objectives and shareholder value. Modified from Walls (1995).

2.4 Objectives under uncertain conditions

The future performance E&P projects is a function of various "states of nature" (e.g., OOIP, reservoir structure) and "states of the world" (e.g., hydrocarbon prices, available technology, political risk).

However, the behaviour of these "states of nature" and "states of the world" is highly uncertain to the eyes of the DM. Hence, the future performance of an investment portfolio formed by E&P projects will be uncertain as well.

This uncertainty implies the existence of a range of possible outcomes in the various performance attributes of the portfolio. If a subset of these outcomes implies a possible monetary loss or unachieved goal, then the portfolio is said to be risky.

If the performance of a given attribute is uncertain but there is a reasonably good understanding of the probability distribution of it outcomes (Bratvold et al. (2002)) then this attribute can be modelled as a probability density function (PDF). Consequently, it is possible to summarise the future performance of an asset or a portfolio in terms of a set of distributions where each distribution corresponds to a particular performance attribute. Newendorp and Schuyler (2000) state that according to utility theory, if the company is risk neutral it is possible to summarize the performance of the NPV attribute with its expected value (EV). However,

Walls and Dyer (1996) have shown that many oil and gas firms are risk averse. If the corporation is risk averse, then the expected value is not enough information to summarise the performance of the NPV attribute and hence the expected value statistic must be accompanied with a risk or uncertainty metric (e.g., semi-standard deviation, standard deviation, probability of achieving a certain target) (Newendorp and Schuyler (2000)).

Similarly, DuBois (2001) shows that risk averse DMs desire to maximise the expected value of the NPV attribute whilst maximising their probabilities of achieving certain goals of other performance attributes (e.g., production, reserves, costs). Additionally the 2000 SPE definition of reserves SPE (2000) defines, in the context of probabilistic methods, proved, possible and probable reserves in terms of percentiles and hence DMs are not only interested in the expected value of this operational attribute.

From all of the above it is possible to state that, under uncertainty, the objectives of a portfolio optimisation problem should not only be set over central tendency statistics (e.g., most likely, expected value) but also over dispersion statistics and probabilities (e.g., semi-standard deviation, standard deviation, percentiles, probability of achieving a certain target). Figure 2-2 shows an example of the hypothetical objectives of a risk averse oil company according to the previous statement. This company is interested in maximising the expected value of the NPV (E(NPV)), minimising the standard deviation of the NPV (SD(NPV)), maximising the expected value of the reserves and maximising the 90th percentile of the reserves[7].

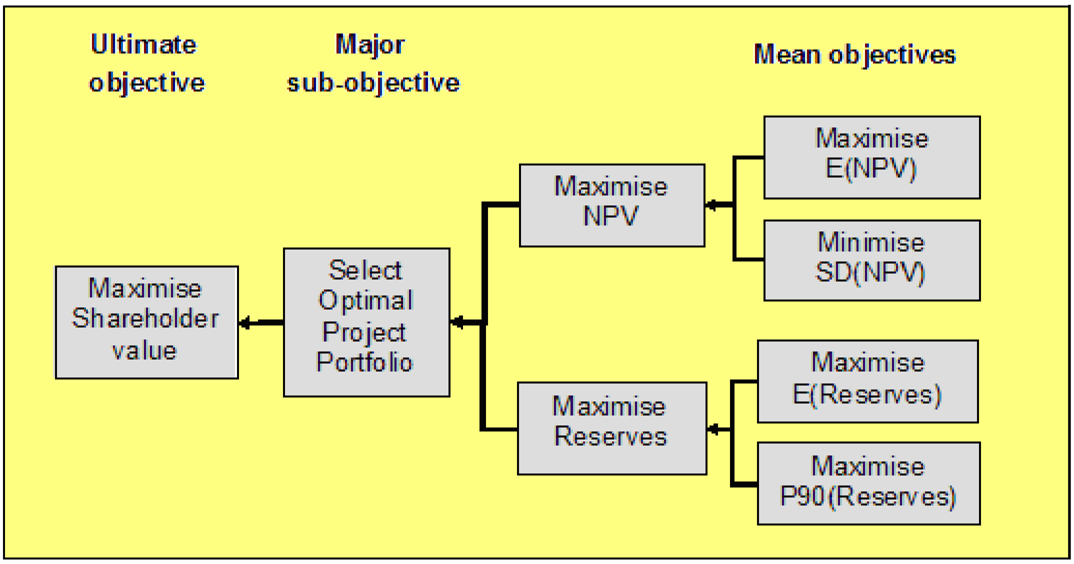

Figure 2-2 Hypothetical mean objectives for a risk averse company.

[7] APPENDIX A shows that the P90 of the reserves is part of the probabilistic version of the SPE definition of proved reserves. Although strictly speaking, these values are not equivalent, the P90 of the reserves attribute could be interpreted as an approximation of the proved reserves value.

2.5 Accounting for correlation

Murtha (2000) states that the correlation between two variables X and Y can be originated in at least three ways. One possibility is that there might be a cause effect association linking the two variables and hence changes in variable X lead to changes in the values of Y. The second possibility is that both X and Y may depend on a third variable, Z. Thirdly, there may be simply a "chance" association between X and Y.

In this thesis, the first two types of correlation will be classified as "structural correlation". This term will be used because the correlation is originated from the structure of the equations of the model. The third type of correlation will be classified as "stochastic correlation". While structural correlation is implicit in the equations of a given model, stochastic correlation has to be explicitly stated.

On the other hand, the decision setting presented here allows classifying correlation using an alternative criteria depending if the correlations occur at the project or at the portfolio level. If the correlation occurs among variables from a single asset, then the correlation is called intra-project correlation. If the correlation occurs among variables from different projects (portfolio level) then the correlation is called inter-project correlation.

2.5.1 Intra-project correlation

The correct characterisation of a portfolio in terms of various performance attributes obviously depends on the correct characterisation of the projects that form the portfolio in terms of those attributes. However, each E&P project is a highly complex system, hence, determining the level of complexity to model these projects is a challenging process itself since changing the complexity of the model impacts the risk and return parameters of the relevant attributes under study (Campbell et al. (2003)).

One issue that clearly affects the complexity of these models is the inclusion of correlation (structural or stochastic) among variables. At an individual project level there are many sources of correlation among the variables that represent the various "states of the world" and "states of nature". Murtha (2000); Campbell et al. (2003) have shown that the inclusion of these correlations have a higher impact on the volatility metrics of the attributes under study (e.g., standard deviation, percentiles) than on the central tendency "return" metrics (e.g., expected value). Therefore, the "tails" of the performance attributes PDFs could be under/over estimated if the correlations are not properly addressed. Since risk is usually associated with the chance of achieving a certain value, then failing to properly characterise the "tails" of these PDFs will clearly impact the assessment of the risk level on a given project.

Correlation can be found at all levels of the E&P system (Begg et al. (2001)). Typical examples of these correlations are the relationship between water saturation and porosity when calculating OOIP at the exploration stage, the relationship between the Capex and the amount of recoverable volumes or the relationship between initial production and the amount of recoverable volumes at the development planning stage.

It is important to note that the performance attributes (i.e., reserves, NPV, cash flow, production) of each project proposal are correlated as well through structural correlation. For example, since the gross revenue is the product of the hydrocarbon price times the production, the performance attribute “gross revenue” will be obviously correlated to the attribute “production” on a given project.

2.5.2 Inter-project correlation

Just as the presence of correlation among variables at the project level may impact the assessment of the volatility of the performance attributes of a project, the correlation among the performance attributes of potential assets has an impact on the volatility of the resulting portfolio. This observation is the root of modern portfolio theory, initially envisaged by Nobel laureate Harry Markowitz.

Markowitz (1952) demonstrated how investors of financial securities could minimize the risk (or standard deviation of historical returns) of their portfolio returns by understanding the correlation among different stocks. This effect, called diversification, is reduced if multiple investments are positively correlated but amplified if the investments are negatively correlated. Moreover, Markowitz (1952) states that there is a combination of securities that maximizes the return for each level of risk. This set called “efficient frontier” is what operation research practitioners call non-dominated solutions or Pareto set[8].

Several authors (Hightower and David (1991); Orman and Duggan (1998); Ball and Savage (1999a); Ball and Savage (1999b); Brashear et al. (2000); Simpson (2002); Bratvold et al. (2003)) have adapted the ideas of Markowitz to the oil and gas project selection arena. However, one of the main hurdles of applying portfolio theory to “real” assets as E&P projects is the multiple sources of inter-project correlation present in the E&P system. Particularly, Ball and Savage (1999a) summarize the main sources of inter-project correlation in five types as follows.

Prices: Oil and gas projects produce these hydrocarbons in various proportions. Oil prices are set in terms of worldwide supply and demand and, as a result, the economic outputs of oil projects are positively correlated relative to fluctuations in crude price. On the other hand, gas prices are mostly set in terms of local supply and demand. As a consequence, gas prices in many parts of the world do not track either world crude oil or each other very well.

[8] These concepts will be extensively discussed in chapter 4.

Places: The operational and financial outcomes of assets in close geographical proximity may be positively correlated through geological similarities or fiscal regimes. Therefore, a portfolio consisting of projects in the same geological site or country would not constitute a highly diversified portfolio.

Profiles: The valuation of oil and gas assets involves the generation of profiles of the relevant operational and financial attributes for each one of the available projects. Often, the more constant these profiles are the better. This observation is particularly important for the cash flow profile, as owning several projects requiring large cash outlays in the same year can be very risky. To deal with this issue, companies may need to delay the starting times for some of their investments, and as a result may need to optimise not only the working interest for each one of the projects, but also their timing.

Politics: Politics can be a source of uncertainty for oil & gas investments. Therefore, projects under the same political regime may be disrupted because of the same political events. The obvious way to diversify this source of risk is to maintain a multi-national portfolio.

Procedures: Technical and managerial procedures may also be a source of statistical dependence. Thus, a firm equipped with specific know-how to explore and produce a particular kind of project will have a positive correlated portfolio by definition.

2.6 Implications of correlation in multi-objective optimisation over several attributes

When the method proposed by Markowitz is used to optimise E&P projects, the objectives are usually set to be the expected value and a dispersion metric (i.e., standard deviation, semi-standard deviation, variance) of the attribute NPV. Therefore, it is only necessary to account for the correlation among the NPVs of the project proposals.

However, when dispersion/risk statistics (standard deviation, percentiles, probabilities of achieving a certain target) are set as objectives in multiple attributes simultaneously (e.g., P90(Reserves), P(NPV>0)) it becomes necessary to account for all the possible inter and intra-project correlations among all the attributes involved. On one hand, as stated at the end of the previous section, the performance attributes of each project proposal are correlated among themselves. On the other hand, the multiple performance attributes of one project proposal may be correlated to the multiple performance attributes of another project proposal. For example, if both projects are oil producers, the "reserves" attribute in one project may be correlated to the "cash flow" attribute in the other project. The reason for this is that both attributes depend on the oil price[9].

2.7 Defining a portfolio: the decision variables

The decision variables are the variables that can be directly modified at the will of the DM. In an oil and gas project portfolio optimisation context, DMs usually need to decide upon the following three issues (Simpson (2002)):

- What are the best projects in which to invest?
- What is the desired working interest for each of the chosen projects?
- When is it best to invest?

Theoretically, DMs may choose any percentage of participation for the project proposals. However, in practice, participation levels must be chosen from the contractual constraints established by the government or the contractor group managing the asset.

Additionally, it is not uncommon that the feasibility of one project might depend on other projects. A typical example of this case in the oil and gas industry can be a small tie back off-shore project that may depend on the processing facility availability of the nearest production hub.

2.8 Performance goals

Another characteristic of the general oil and gas portfolio optimisation problem is that corporations usually manage the multi-objective quality of the problem by setting goals on multiple performance metrics. These goals can be set on a "total" cumulative basis (i.e., cumulative production over the entire life of the asset) or on the yearly performance during a number of years into the future (e.g., cash flow in the years 2012, 2013 and 2014). It is important to note that since goals are either achieved or not, they must be included in the constraints of the portfolio optimisation statement, not in the objective function[10].

These goals can broadly be classified in two types (Simpson (2002)). The first type is of strategic nature and represents the aspirations of the DMs of the corporation. These goals can be set as minimum production requirements in certain years, the total amount of reserves added in a certain period and/or the focus on a given geographical area or technology. However, although modelled as constraints, these goals are not real commercial or physical constraints. The purpose of these goals is to achieve long-run profitability and stability Levy and Sarnat (1994). The second type of goals represents real

[9] The "reserves" attribute depends indirectly on the oil price since the production is a function of the economic limit of the project, and the economic limit is a function of the oil price. The economic limit concept will be discussed in chapter 6.
[10] The structure of an optimisation problem (or mathematical program) will be explained in chapter 4.

constraints, they are based on contractual obligations, or the availability of a certain type of resource (e.g., rigs, capital).

In other words, the first set of constraints represents the world "how DMs want it to be" while the second type of constraints represents the world "how it is". In regard to the first type of goals it is relevant to note that there is not a clear procedure in the literature to assure that the aspirations of corporate planners are in line with the reality of their investment portfolios. Hence, the optimisation may not be able to find a feasible solution. Typically, this is explained by conflicting goals, an inappropriate set of projects or a combination of the two. In these cases, corporate DMs may decide to restate the goals of the company and/or promote the search for new opportunities to be valuated and incorporated to the portfolio optimisation process in an iterative basis until a feasible solution is found.

The previous point is of special relevance for this thesis. It will be shown in chapter 7, that the tradeoffs among performance attributes can be complex, even for a small simple set of project proposals. Hence, setting these goals without a clear understating of these tradeoffs can prevent DMs from discovering the real potential of the portfolio. In other words, the fact that a feasible solution satisfies the preferences of the DMs is found do not mean that is the solution that will deliver the highest level of perceived utility.

2.9 Summing up

This chapter described the main features of an oil and gas project portfolio decision setting. These characteristics are:

- Limited resources (money, people, equipment) to pursue every available project.
- Uncertain performance of attributes.
- Oil and gas companies are generally risk averse.
- Presence of intra-project and inter-project correlation.
- Two decision variables: working interest and time to invest.
- Projects that depend on the presence of other projects.
- DMs typically address the multi-objective quality of the portfolio optimisation problem through goals.

The model presented in this thesis addresses all the previous characteristics of the general upstream oil and gas optimisation problem with the exception of using "time to invest" as a decision variable and the fact that a project may depend on the presence of another project. The reason for this is that the optimisation algorithm used in this thesis assumes that the decision variables are continuous and linearly constrained. Consequently, adding the "time to invest" decision variable would imply the use of an integer variable because the timing of these projects is usually modelled in a yearly basis.

Additionally, the dependency on the presence of other projects would also imply the need for integer variables and/or “if” statements that would break the assumption of linear constraints.

3 OIL & GAS PROJECT VALUATION: A MULTI-ATTRIBUTE PERSPECTIVE

3.1 Introduction

The purpose of this chapter is to present a theoretical background to characterise and valuate E&P investments from a multi-attribute perspective whilst accounting for uncertainty and risk. The first part of the chapter introduces the stochastic characterisation of E&P projects in terms of multiple operational and economical performance attributes. The second part introduces concepts from single attribute and multi-attribute utility theory to explain the need to include the preferences of the DM in the presence of multiple attributes and risk aversion. Lastly the chapter introduces the need for risk measures as a result of the difficulties to implement utility theory in practice.

3.2 Project valuation

To maximise the shareholder value of oil & gas upstream companies DMs require various yardsticks for measuring the value of their investment portfolio. Since individual project proposals are the building blocks of an investment portfolio, in order to measure the value of a given portfolio it is necessary to firstly characterise the individual project proposals in terms of these various yardsticks.

The number of project performance attributes (yardsticks) shown in the literature is vast (Remer and Nieto (1995a); Remer and Nieto (1995b)). However, Pande (2003) states that the attributes that have more impact on the value perceived by shareholders of upstream petroleum companies are **cash flow, reserves** and **production**.

The performance of a given E&P project on these attributes is highly uncertain on its initial stages. This uncertainty reduces as the project evolves in time. Hence, it is only possible to know the performance of a given project with certainty at the end of its life, many years after the investment decision was made.

The major consequence of exposing large investment capital to this initial uncertainty is that projects may perform poorly on relevant attributes. This implies a potential for losing the invested resources. For this reason upstream oil and gas projects are said to be risky. In order to address the uncertain and risky qualities of these projects, DMs rely on mathematical models to characterise the future performance of E&P projects in terms of a relevant set of performance attributes.

3.3 Mathematical modelling

Mathematical models allow DMs to analyse various decision alternatives before having to choose a specific plan for implementation. Bender (1991) defines a mathematical model as “an abstract, simplified, mathematical construct related to a part of reality and created for a particular purpose”.

Although it is obvious that the main objective of a model is to obtain an adequate representation of the underlying problem, when working with complex systems as E&P projects, it is necessary to also account for additional objectives as solution time and model size (Lund (1997)). In other words, the models should be solvable within a reasonable time frame and a reasonable amount of computer power.

The inputs of these mathematical models of E&P projects are the various “states-of-nature” and “states of the world” previously mentioned. The “states of nature” are variables mostly related to the geological and physical characteristics of the projects considered. The “states of the world” are variables mostly related to the available technology, market conditions and the future prices of the hydrocarbons contained in the assets. The actual model is a set of equations that relate all these variables. This set of equations describes the behaviour of the upstream oil and gas system.

3.4 Deterministic and probabilistic approaches to modelling and valuation

Once the set of equations describing the project is defined, there are two possible approaches to calculate the performance attributes, deterministic or probabilistic. The deterministic approach uses single “best” estimates of the input variables of the set of equations. Consequently, this approach also results in a single “best” estimate of the future performance of one or several performance attributes. While useful, the approach neglects the uncertainty in the performance attributes and, therefore, provides no indication of the associated risks of the investment.

It is important to note that it is a common practice to use the expected value of the input variables of the system as a “best” estimate of the performance of that variable. However, if the system of equations is not linear, inserting the expected values of the input variables on the system of equations will not deliver the “true” expected value of the attribute (Begg et al. (2004)).

In contrast, in the probabilistic approach the key input variables are defined as probability-density functions (PDF) instead of using single “best” estimates. These PDFs are set in a Monte Carlo simulator to calculate probability distributions for the relevant performance attributes. Monte Carlo simulation is a technique that repeatedly generates scenarios driven by randomly sampling the input probability distributions. Each of these repetitive calculations of the output distributions (performance attributes) is called a trial. If a large number of trials are performed, the shape of the inputs distributions is preserved and it is possible to generate an approximation of the output distributions.

Performing probabilistic analysis of projects with Monte Carlo simulation is clearly a superior approach to project valuation than the deterministic one. The main reasons to state this is that it not only provides a complete distribution of the possible outcomes of the project and hence it accounts for the possible risks of the project, but also provides a robust way to calculate the expected value of an output distribution regardless of the non-linearities of the system.

However, the output distributions of a Monte Carlo simulation strongly depend on the shape of the input distributions, and hence it is necessary to rely on expert opinion and/or historical data to define the shape of these distributions. This issue could sometimes become a problem since historical data may not exist or experts may not be trained to think probabilistically. Another problematic area of Monte Carlo simulation is that there might be correlation among the input variables and finding the appropriate correlation factor to model these dependencies is not necessarily straightforward.

3.5 A general framework to model and characterise upstream projects stochastically

A detailed explanation of the upstream oil and gas system goes far beyond the scope of this research. It is thus necessary to make simplifying assumptions to allow attention to be focussed on the areas of the system that have more relevance to this research. As a result, this thesis will divide the system in three major stages: **resources estimation**, **production prediction** and **economic valuation**. This section describes, for each one of these stages, the main variables that must be addressed to build a mathematical model of an oil producer upstream project from a high level perspective.

3.5.1 Resources estimation

The purpose of this stage is to estimate the amount of petroleum that it is possible to extract from the ground. The uncertainty associated with the estimation of these volumes has led the oil and gas industry to develop a jargon to classify resources and reserves that is both vast and ambiguous. However, two concepts of particular relevance to estimate the amount of recoverable volumes are the **discovered petroleum-initially-in-place** and the **estimated ultimate recovery** [11].

The **discovered petroleum-initially-in-place** accounts for the total volume of oil that is expected to be contained in a given reservoir. For the purposes of this research this concept will be assumed to be equal to the original oil in place (OOIP) that can be calculated with the following equation[12]:

$$OOIP = 7758Ah\phi(1 - S_w) / B_0$$

Each one of the variables in this equation is uncertain and hence can be modelled as a probability distribution. Then, a Monte Carlo simulation can be performed to estimate a resulting distribution for the OOIP. According to the central limit theorem, a stochastic variable that is equal to the product of other

[11] The SPE definitions and terminology used for these terms are described in the appendix 1.
[12] See appendix 2 for a description of the variables and units of this equation.

stochastic variables tends to exhibit a log-normal behaviour (Murtha (2000)). As a consequence, the OOIP is usually modelled as a lognormal distribution.

The **estimated ultimate recovery** (EUR) is a subset of the "discovered petroleum-initially-in-place" that is **technically** feasible to produce. Equation 3-1 shows that this volume can be calculated multiplying the OOIP by a recovery factor (RF). The RF is another uncertain variable that represents the fraction of the OOIP that is possible to recover from the reservoir of interest. However, this amount of hydrocarbon that is technically feasible to recover is considered to be part of the **resources** of the company and is not part of the **reserves** of the company. The reason for this is that the term "reserves" is used for the amount of resources that are **economically** feasible to produce.

$$EUR = OOIPxRF \qquad \textbf{Equation 3-1}$$

3.5.2 Production estimation

This stage mainly involves the derivation of a yearly hydrocarbon production profile. This profile intends to show the timing in which the EUR is supposed to be depleted from the reservoir. As a consequence of the uncertain quality of the EUR, it is convenient to model this production profile stochastically using Monte Carlo simulation (Murtha (2000)). In other words, the production in each time period (i.e., days, months, years) will be a probability distribution as well.

In addition to the production profile, this stage also defines the capital expenses (Capex) and operating expenses (Opex) of the project under evaluation. These metrics are also uncertain because they partially depend on information that is revealed while the field is being produced (i.e., more wells may be needed and hence a larger capital would be necessary). As a result, these costs can also be modelled as probability density functions. However, once a clear development plan has been set for the project, most of the uncertainty in the Capex is reduced and hence modelling the Capex as a deterministic variable could be reasonable under the assumption that potential expansions of the current project (i.e., drill more wells) needing a non-marginal incremental capital could be considered new project proposals.

3.5.3 Economics

The main purpose of this stage is to produce a yearly after tax net cash flow profile. Most of the complexity of this stage derives from the fact that although geological, engineering and financial concepts are universal, most of the fiscal terms that rule the exploitation of oil and gas reservoirs are set locally (Johnston (1994)). As a consequence, the after tax cash flows of two hypothetic geologically identical projects located in different countries may differ vastly.

For the illustrative purposes of this section a simple "generic" concessionary regime will be assumed. Hence, the after tax net cash flow (ATNCF) for a given year is calculated multiplying the yearly production times the forecasted price of the hydrocarbons for that year and subtracting the costs derived from the previous stage and the royalties and income taxes. The uncertain quality of hydrocarbon prices may also be modelled as a stochastic process where in each time period the oil price is a probability distribution. The resulting after tax net cash flow would be as follows:

$$ATNCF_t = \left(Q_t \cdot P^{oil}{}_t\right) - CAPEX_t - OPEX_t - Tax_t - ABEX_{EconLimit} \qquad \textbf{Equation 3-2}$$

Where for a given year t, Q_t is the oil production, $P^{oil}{}_t$ is the oil price[13] and Tax_t is the total income tax plus royalties. The $Abex_{EconLimit}$ term represents the abandonment expenditures associated with the investment at its economic limit. Since most of these parameters are uncertain, the resulting ATNCF profile could also be modelled as a stochastic process and hence its yearly performance would be a PDF. Then, the cash flow profile can be discounted by a given rate to obtain a PDF of net present value (NPV) attribute. This rate used to discount the cash flow profile accounts for the opportunity cost of putting the capital in alternative investments (i.e., other projects, financial assets) (Brealey and Myers (2000)).

The after tax cash flow profile stage has a strong interaction with the production profile stage. The main reason for this is that although it is possible to make a forecast of the production from a technical point of view, the production of the field is terminated at its economic limit ($t_{EconLimit}$) which is the point in time where the project becomes uneconomical (i.e., the operating cash flow of the project becomes negative).

The reserves of the project are defined as a subset of the "discovered petroleum-initially-in-place" that is considered to be technically feasible and economically viable to produce. Therefore, it is only possible to calculate the reserves of the project once the economic limit of the project is calculated. Hence, once the economical criterion to close the field has been defined it is possible to calculate the reserves (R) of the project as follows:

$$R = Q_{EconLimit} = \sum_{t=0}^{t_{EconLimit}} q_t$$

Since the addition of several stochastic variables will be another stochastic variable, the attribute reserves will be a probability distribution. When this probability distribution is plotted in its cumulative form is commonly called "expectation curve" (Behrenbruch (2004)). The SPE classification of reserves

[13] The next section describes one of the possible methods to model the stochastic behavior of oil prices.

includes, among other technical and economical factors, the following probabilities when stochastic methods are used[14]:

- **Proved:** better than 90% chance of being recovered.
- **Proved plus probable**: better than 50% chance of being recovered.
- **Proved plus probable plus possible:** better than 10% chance of being recovered.

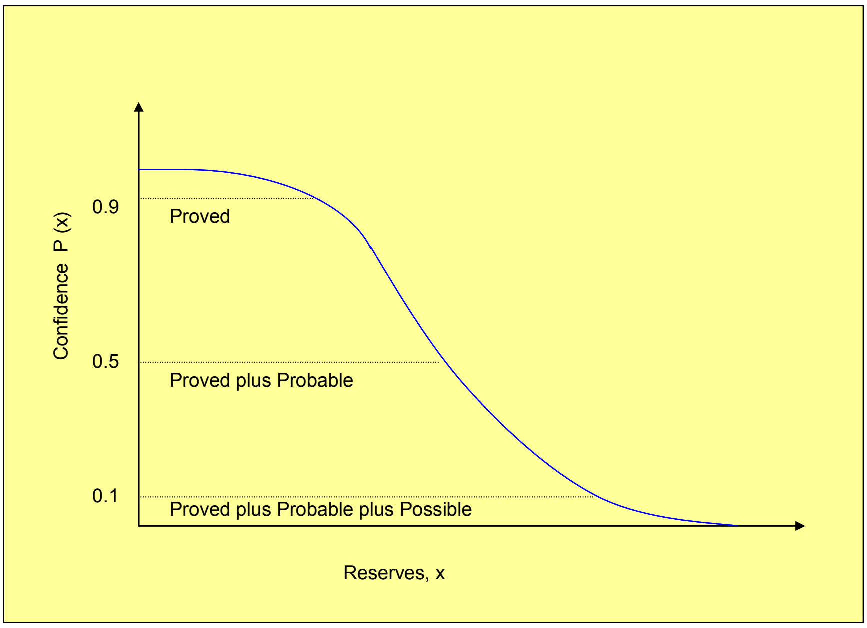

Figure 3-1 Distribution of Reserves.

3.5.4 Stochastic oil price modelling

Equation 3-2 shows that the income is generated through the sale of oil. Consequently, the overall economy of the field strongly depends on the price of the oil. Therefore, an assessment of possible future oil price paths is inevitable to get a realistic forecast of the performance a given project. Since the future behaviour of oil prices is uncertain it is reasonable to think of oil prices as stochastic processes. The objective of a stochastic oil price model is not to predict the actual price at a particular time in the future. Rather, a stochastic oil price model attempts to capture the characteristics of the way it fluctuates with time.

Lund (1997) shows that the two more common approaches to do so are the Brownian motion and the mean-reverting models. However, Dixit and Pindyck (1994) argue that the mean reverting stochastic model is the one that best resembles the historic behaviour of the oil price. The basic idea behind this model is that if the oil price is too high above or to low below a certain long run equilibrium level, the market forces will act to respectively reduce or increase the supply of oil. This creates a mean-reverting

[14] The SPE definitions and terminology used to classify reserves are described in the appendix 1. It is relevant to note that, in a strict sense, the P10, P50 and P90 values do not correspond to the Proved, Proved plus Probable and Proved plus Probable plus Possible SPE definitions, but could be used as rough approximations of these.

process that acts like an elastic band, the further the price is from this long term mean, the stronger its tendency to revert to its long term equilibrium level.

The oil price model used in this thesis is a standard mean-reverting stochastic process (Dixit and Pindyck (1994)) that is defined by the following equation:

$$p_t = p_{t-1} + (M - p_{t-1}) \cdot \Delta t \cdot \eta + \sigma \cdot \varepsilon_t \cdot \sqrt{\Delta t}$$

Where:

t = time period, years

p_t = oil price at time t, $

M= long term mean oil price, $

$\Delta t = t_t - t_{t-1}$ = time increment, years

η=speed of oil price reversion to long term mean

σ=standard deviation of oil price annual increments

ε_τ= standard normal random variable with mean equal to 0 and SD equal to 1

In this manner, the price is a Markov process as the price at any time is calculated from the previous price. Additionally, the price at any time t is added to a factor that depends on the reversion property, plus a random increment. An absolute minimum value of the oil price is also specified on the basis that market forces would prevent the price going below this minimum. Figure 3-2 shows how the curve starting at P_0 represents the expected value of P_t which decreases as time evolves if $P_0 > M$ and the curve converges towards the equilibrium level M. In the same figure, note that the variance increases until a certain time and then remains constant. This is a result of the mean-reversion force effect that, even in a distant future, prevents the values of P from being too far from M.

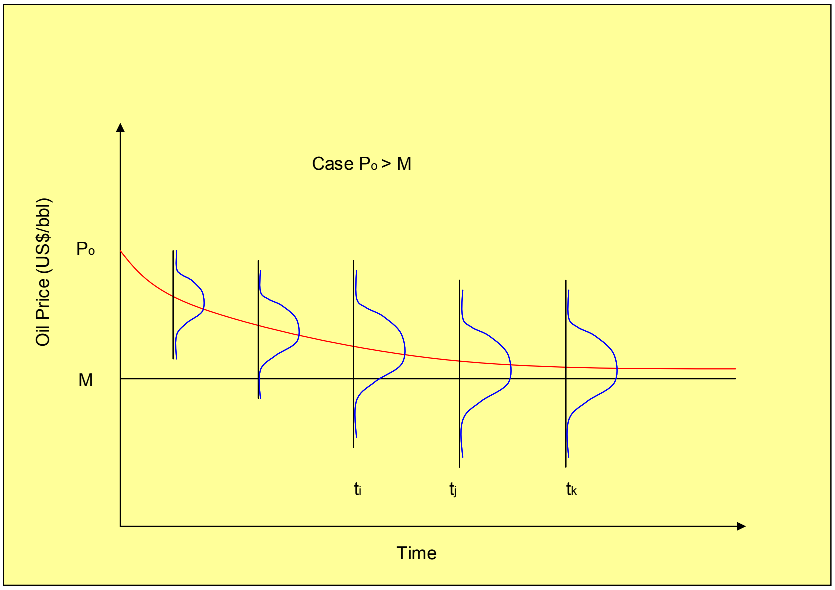

Figure 3-2 Mean-reverting price forecast model. Modified from Dias (2004)

3.6 Valuation

Once a project or a portfolio of projects has been characterised[15] in terms of the various performance attributes described in section 3.5, corporate DMs need to assign an overall value to it. This process is called valuation, and usually involves assigning a **monetary** value to the investment under study.

Classical finance theory (Brealey and Myers (2000)) states that the expected value of the net present value of the investment (E(NPV)) should be the only decision criteria needed to value an investment. Hence, in order to maximise shareholder value, it is necessary to maximise the expected value of the net present value of its investment portfolio. However, this statement assumes that:

- The corporation has access to unlimited funds.
- The corporation attitude towards risk is neutral.
- Shareholders perception of the value of the company is not a function of other performance attributes than NPV.

When the first assumption does not hold, finance theory (Brealey and Myers (2000)) suggest that DM must base their decisions on a capital efficiency measure as the expected net present value over investment ratio (E(NPV)/Inv)). This ratio adjusts the expected value of the NPV for the size of the discounted investment over the life of the project (Inv).

However, finance theory does not prescribe clear paths of action when the statements in the assumptions 2 and 3 do not hold. The vast field of utility theory and multi-attribute utility theory provides

[15] The process to characterise an E&P portfolio from the calculated performance attributes of individual projects is in chapter 5.

a logical and consistent way to articulate investment decision problems where decision makers are non-risk neutral and NPV is not the only appropriate attribute.

3.6.1 Utility theory: one attribute with uncertain performance

Utility theory was formalized by von Neumann and Morgenstern (1944) and provides a systematic way to rank alternatives while accounting for the risk aversion of the DM(s). Walls (1995) states that, according to this theory, when an oil and gas corporation is not risk neutral but is interested in the performance of a single attribute (i.e., NPV) the value of a given investment opportunity should depend on:

- The likelihood of the possible outcomes of the attribute under study if the investment is taken.
- The preferences of the DM towards those possible outcomes.

However, it is important to note that including the risk preferences of the DM in the valuation process radically changes the meaning of the term “value”. From a utility theory point of view the term “value” no longer means value in monetary terms but rather a level of satisfaction in accordance to the preferences of the DM(s). Hence, the term “utility” is preferred to the term “value” in this context. Since a utility function is used to provide a ranking among investment alternatives its absolute cardinal value (utility) has no meaning (Luenberger (1998)) and all that matters is how it ranks mutually excusive alternatives when their expected utilities are calculated.

In this manner, the relationship between the corporate risk preferences and the consequences of an investment decision can be described through a function called “utility function” (Schuyler (2001)). This function relates the performance of a given decision alternative (i.e., investment) on a given attribute (i.e., NPV) to the utility perceived by the DM.

The specific shape of a utility function varies among individuals and corporations. The utility function u(x) is a monotonically increasing curve where the vertical axis (utility) increases in numerical value as the performance on the attribute (x) under study increases. At least theoretically it is possible to draw just such a curve for any individual or organization.

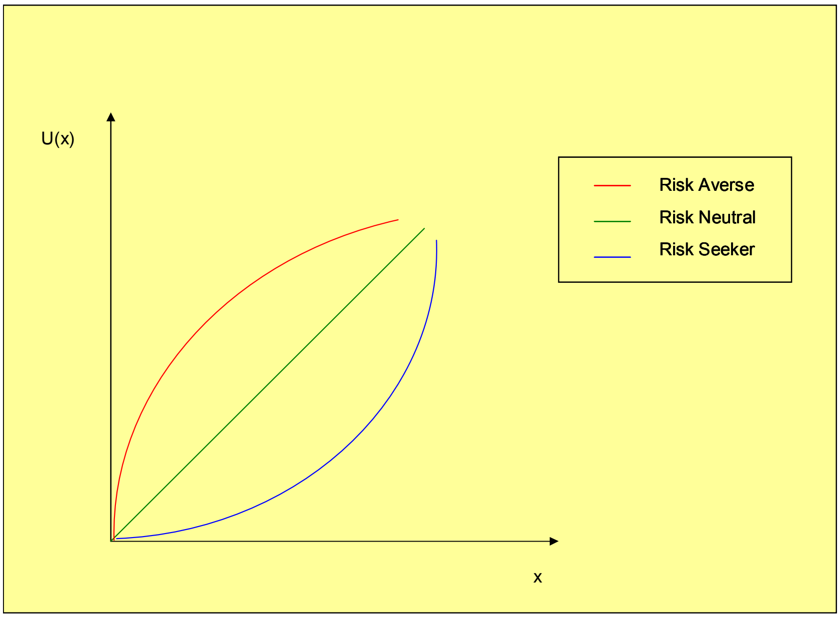

Figure 3-3 Single attribute utility function.

The likelihood of occurrence of the possible consequences of each alternative is included in the analysis with the use of the expected perceived utility E(u(x)) as follows:

$$E(u(x)) = \sum_{j=1}^{n} p_j u(x_j)$$

Where x is the attribute under study, p_j is the probability of outcome j so that $\sum_{j=1}^{n} p_j = 1$. In this manner, the value added by the investment for its performance on the attribute x would be equal to the **expected utility** of the attribute. Hence, according to this theory, in order to maximise value corporate DM must chose the investments with higher expected utility.

The risk attitude of the corporation is included in the utility curve through its shape (see Figure 3-3). If the function is convex, the corporation is said to be **risk averse**. If the function is concave, the corporation is said to be **risk seeker**. If the function is linear the corporation is said to be **risk neutral.**

When the corporation is risk neutral it is possible to base corporate decisions solely on the expected value of the attribute under study. If u(x) is a linear function of positive slope then:

$u(x_i) = bx_i + c$ and,

$E(u(x_i)) = E(bx_i + c) = bE(x_i) + c$

Where b is a positive constant. From this equation it is possible to infer that if corporate DMs want to maximise the expected utility of a risk neutral corporation, they only need to maximise the expected value of the attribute under study. Hence, it is not necessary to elicit a utility function from the DMs to convert the forecasted performance of the attribute into utility units and then calculated the expected utility. This demonstrates why finance theory states that risk neutral DMs should solely base their decisions on the expected value of the NPV.

However, Walls and Dyer (1996); Schuyler (2001) show that very few oil corporations and individuals are impartial to the potential profit and losses encountered in the upstream oil business. Moreover, these authors emphasize that most upstream petroleum companies show risk averse behaviour. As a result, in most upstream petroleum oil companies, the expected value of the attribute under study does not provide sufficient information to make an investment decision. Consequently, according to utility theory, most oil and gas companies should explicitly articulate their utility functions in order to systematically account for their risk preferences in their investment decisions.

3.6.2 Multi-attribute utility theory

The previous section described a systematic approach to compare among investment alternatives whilst accounting for the risk attitudes of DMs but accounting for a single attribute. Multi-attribute utility theory is the extension of this approach to decision situations where the future performance of a decision is measured in terms of several attributes. A thorough description of multi-attribute utility theory can be found in (Keeney and Raiffa (1993)).

Multi-attribute utility theory states that if the condition of **utility independence** holds among all the relevant attributes (Clemen and Reilly (2001)), it is possible to generate individual utility functions for each of the attributes and then aggregate these individual functions into one function. In this manner, the overall expected utility of a decision becomes a function of the individual expected utility of each one of the relevant attributes. However, the theory states that individual utility functions must be "normalised" before being used as an input to the aggregate function. Hence, each utility function must be recalculated to assign values of 0 and 1 to the worst and best levels on that particular attribute. This can be represented mathematically for a two attribute (x_1,x_2) problem as:

$$E(u(x_{1,}x_2)) = f[E(u_1(x_1)), E(u_2(x_2))]$$

Where $E(u(x_1,x_2))$ is the overall expected utility and $E(u(x_1))$ and $E(u(x_1))$ are the expected utilities of attributes x_1 and x_2 respectively. However, as stated before, doing this is only possible if the condition of utility independence holds. This condition essentially states that the shape of the individual utility functions of each of the attributes under study must be independent of the actual outcomes of the decision in the other attributes. In other words, the risk preferences of the DM towards the attribute x_1 should not be influenced by the future value of attribute x_2.

The simplest way to aggregate individual utility functions is by using an additive utility function (Clemen and Reilly (2001)). Using a two attribute decision setting as an example, the additive utility function can be represented mathematically as:

$$E(u(x_{1,}x_2)) = k_1 E(u_1(x_1)) + k_2 E(u_2(x_2))$$

Where k_1 and k_2 are the weights that the DM must assign to each attribute in accordance to their relative importance to the decision so that k_1 and k_2 add up to 1. However, it requires an even stronger condition than utility dependence called **additive independence (**Clemen and Reilly (2001)). Fundamentally, the additive independence assumption states that to use an additive utility function it is necessary that the individual utility functions of the attributes under study are mutually independent. Although similar, this assumption differs from the utility independence assumption in that changes in the utility function of one attribute affects the shapes of the utility functions of the other attributes while the utility independence assumption state that **actual** outcomes (sure levels) in one attribute affects the shape of utility function of the other attributes.

It is possible to note that although this approach is very straightforward to implement if the additive assumption holds, the assessment of the DM preferences to check for this assumption could become a very complex exercise. Additionally, this approach misses the fact that the utility that a DM assigns to the overall performance of the investment may be also influenced by the interactions among the expected utilities of attributes under consideration. Clemen and Reilly (2001) also states that the additive independence assumption rarely holds, hence the use of the additive is mostly recommended for decision situations where there is little or nil uncertainty.

As a result, multi-attribute utility theory proposes the use of a multiplicative utility function to account for interactions among attributes whilst overcoming the need for the additive independence assumption (Clemen and Reilly (2001)). Using a two attribute decision setting as an example, the multiplicative utility function can be represented mathematically as:

$$E(u(x_{1,}x_2)) = k_1 E(u_1(x_1)) + k_2 E(u_2(x_2)) + (1 - k_1 - k_2) E(u_1(x_1)) E(u_2(x_2))$$

Where the term: $(1 - k_1 - k_2) E(u_1(x_1)) E(u_2(x_2))$, accounts for the interaction among the attributes.

It is possible to generalize the previous equation to for n different attributes. However, Clemen and Reilly (2001) states that when there are three or more attributes modelling preferences is more difficult and, therefore, building a utility function that will permit interactions across many attributes can become complex. Additionally, when the utility independence assumption does not hold it is not possible to aggregate the individual utility functions of the attributes under study and therefore the assessment of the multi-attribute utility function becomes an even more complex exercise.

3.6.2.1 Risk measures

Last section showed that utility theory provides a systematic way to rank alternatives while accounting for the risk aversion of the DM(s) and multiple attributes. However, its actual application to solve real life problems has been heavily criticized by many finance authors (Roy (1952); Nawrocki (1999)). The major source of criticism is the practical difficulty to build a utility function that truly represents the preferences of a DM.

In the upstream petroleum business, Newendorp and Schuyler (2000) states that the use of utility theory for risk policy is scarce. Similarly, these authors also state that one of the possible explanations for this is that decision analysts continue to struggle with accurately describing a decision maker's risk preferences.

In the seminal paper of portfolio theory Markowitz (1952) shows that an alternative approach to utility theory is to explicitly include a risk metric in addition to the expected value of the attribute of interest. This approach suggests that instead of having to elicit a utility function, a DM may rather make a direct judgement about the value of the investment by trading-off his risk-return preferences. In this manner the DM can express her preferences without the need to articulate them explicitly.

In this approach, the return and the risk of the investment are defined as its expected value and its standard deviation (or variance) respectively. Both statistics are calculated from the distribution of possible results of the attribute of interest. However, to the knowledge of the author of this thesis, the literature proposing the extension of the risk-return concept from one to several attributes in the project selection context is scarce, being Graves and Ringuest (2003) and Medaglia (2003) among the few.

On the other hand, although the concept of characterising an investment in terms of its expected value and a risk metric is widely accepted in the finance and upstream oil and gas literature, up to date many questions are still open at this point regarding which is the correct risk metric to use.

The critics of the use of standard deviation (or variance) as a risk measure argue that risk averse individuals (or corporations) usually avoid outcomes below a certain target level, not the overall uncertainty in the outcomes. Roy (1952) named this fact as the "safety first principle". This author states that in the economic world, disasters may occur if an individual makes a net loss as a result of some investment activity. Hence the safety first principle asserts that in practice, it is reasonable that a DM will seek to reduce as far as is possible the chance of a disaster. However, for an investment with non-symmetrical return distributions, which is the case of most upstream projects, the minimisation of the standard deviation penalizes projects with upside as well as downside potential. In other words it may reduce the chance of a disaster but also reduces the chances of a high positive performance.

The family of the so called "downside" risk measures has been proposed in the literature to account for this issue. Nawrocki (1999) provides a thorough review of the evolution of these measures and summarizes the main factors affecting the choice of the risk measure as follows:

1. Investors perceive risk in terms of below-target returns.
2. Investors' risk aversion increases with the magnitude of the probability of ruinous losses.
3. Investors are not static. As the investor's expectations, total wealth, and investment horizon change, the investor's below-target return risk aversion changes. Investors have to be constantly monitored for changes in their level of risk aversion.

It is possible to infer from factors 1 and 2 that DMs not only want to maximise the probability of achieving a certain target (e.g., P(NPV>0)) but are also concerned with minimising the magnitude of potential losses. In other words, given two possible investments with the same expected value and the same probability of achieving a certain target, a risk averse DM will chose the investment with the lowest chance of a ruinous loss.

On the other hand, the 3rd factor mentioned by Nawrocki (1999) makes it possible to infer that if a DM wants to use a utility function to rank investments according to their expected utility this utility function should be continuously revised to reflect changes in his level of risk aversion.

McVean (2000) reviews the impact of using different risk measures in the construction of efficient portfolios[16] of upstream projects. In this paper the author uses the risk measures most used in the upstream petroleum decision analysis literature. These measures are:

- Standard deviation
- Semi-standard deviation
- Fixed percentiles
- Probability of achieving a given target value.

McVean (2000) shows that the usage of different risk measures drastically affects the results of an oil and gas project portfolio optimisation problem. The author concludes that the use of multiple risk measures can help the DM to gain insight about the potential of an investment portfolio.

[16] The efficiency concept will be discussed in the next chapter.

3.7 Summing up and discussion

The first part of this chapter showed that it is possible to characterise the full life cycle E&P project proposals in terms of multiple operational and economical attributes whilst accounting for uncertainty with the use of Monte Carlo simulation. It is important to note that Simpson et al. (2000) shows that this type of methodology is not widely applied in the day-to-day practice of the oil industry. However, the same authors show evidence that the use of this type of methodology seems to be correlated to the business success of oil companies.

The second part of this chapter showed that financial theory does not prescribe clear paths of action to valuate investments in the presence of multiple attributes and/or risk averse DMs. Utility theory was briefly introduced to show that when DMs are not risk neutral and care about the performance of an investment in terms of multiple attributes, the value (utility) of that investment may differ among different DMs with different preferences. Hence, it is necessary to somehow include these preferences in a valuation methodology with these characteristics.

The second part also showed that the derivation of single and multiple attributes utility functions, although theoretically strong, can become a cumbersome exercise. It was also shown that an alternative approach to the use of utility functions is to calculate a set of non-dominated solutions for the DM and then let her express her risk-return preferences without the need to assess a utility function. It also addressed the fact that this latter methodology has two main flaws, the first being the lack of agreement in the literature to which is the proper risk measure to use and the second the lack of theoretical background to use this type of technique when the decision setting requires multiple attributes.

Taking in account the contributions of Nawrocki (1999) and McVean (2000) seems reasonable to state that to adequately address the risk of an investment it would be necessary to break it down into components. In other words, it should be possible to characterise the risk of an investment with several "risk metrics". If DMs not only want to minimise "below target" probabilities but also want to minimise the magnitude of disastrous events whilst maximising the "upside" potential of an investment, it would be necessary to define "risk" as a multi-objective problem itself. This statement will be considered in chapter 7.

4 OPTIMISATION

4.1 Introduction

The purpose of this chapter is to present a review of the main mathematical programming concepts. Firstly, a general single objective optimisation problem is described. Then, a general optimisation problem with multiple objectives is defined and the main differences between this problem and the single objective one are highlighted. The necessity to include the DM preferences in a multi-objective problem before or after the optimisation is also addressed. Finally, the multi-objective genetic algorithm with linear constraints (MOGOL) selected to act as a search engine in the E&P project selection model presented in this thesis is described.

4.2 Optimisation definition

Optimisation is a field of management science also called mathematical programming that deals with finding one or more feasible solutions that correspond to extreme values of one or more objectives (Deb (2001)). In business decision-making, optimisation methods are of great importance as they help DMs to find investment strategies that provide the “best” performance on one or several corporate objectives given a limited amount of capital and non-capital resources. Generally, optimisation problems (i.e., mathematical programs) are stated in terms of an objective function, a set of decision variables and a set of constraints.

The **objective function** is a function that the DM wants to optimise (minimise or maximise). The set of **decision variables** are the variables that affect the value of the objective function. These set of variables represent a decision to be made and, hence, are the quantities that the optimisation method has to determine. The set of **constraints** allows the decision variables to take on certain values but exclude others. Additionally, many optimisation problems have a special set of constraints that require the decision variables to be nonnegative.

4.3 Single objective optimisation

The general single-objective mathematical program with the objective function **Z(x)** with n decision variables, x_n, and m constraints, $e_m(\mathbf{x})$, may be defined mathematically as:

Optimise: $$Z(x_1, x_2, \ldots, x_n)$$

Subject to: $$e_m(x_1, x_2, \ldots, x_n) \le 0; \ \forall\, m = 1,2,\ldots,M;$$

$$x_n \ge 0; \ \forall\, n = 1,2,\ldots,N;$$

Or in vector notation:

Optimise: $\mathbf{Z(x)}$

Subject to: $\mathbf{e(x) \leq 0}$

$\mathbf{x \geq 0}$

4.4 Multiple objective optimisation

The task of finding one or more optimum solutions when an optimisation problem involves more than one objective function is called multi-objective optimisation. Within a multiple objective context, a DM considers a set A of alternatives and seeks to choose an "optimal" alternative considering all the attributes that are relevant to the analysis. Formally, an **attribute**[17] g_i is a non-decreasing real-valued function that describes an aspect of the global performance of the alternatives. On the other hand, an **objective** defines a direction in which the DM strives to perform better on a given attribute. If a DM has the objective of maximising the performance of attribute g_i (i.e., prefers more of attribute g_i than less of attribute g_i), the alternatives are compared to each other as follows:

$$g_{ji} > g_{ki} \Leftrightarrow \mathbf{a_j} \succ \mathbf{a_k} \quad (\mathbf{a}_j \text{ is preferred to } \mathbf{a_k})$$

$$g_{ji} = g_{ki} \Leftrightarrow \mathbf{a_j} \sim \mathbf{a_k} \quad (\mathbf{a}_j \text{ is indifferent to } \mathbf{a_k})$$

Where g_{ji} denotes the performance of alternative $\mathbf{a}_j$ on criterion g_i. Since the performance of the alternatives on the various attributes will usually lead to conflicting results and conclusions, the "optimal" alternative is not optimal in the traditional single objective optimisation sense. Instead, it is a satisfactory non-dominated[18] alternative (i.e., an alternative that is in accordance with the value system of the DM and is not dominated by other possible alternatives) (Zopounidis and Doumpos (2002)).

After Markowitz (1952), a bi-objective form of this approach became widely known among financial markets specialists for the two objective situation where it is possible to generate an efficient frontier of non-dominated alternatives in order to trade-off risk against return before choosing a final solution[19]. As a consequence of this, Steuer (1985) states that "what is not well known is how to address investment problems where three or more criteria exist". The main obstacle encountered to transit from two to more than two criteria is that the efficient frontier is no longer a frontier but becomes a surface. This then

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[17] The concepts of criteria and attribute will be used indistinctively in this thesis.

[18] If there is an alternative "A" that is at least as preferred as "B" for each one of the attributes that are being considered, and if "A" is strictly preferred to "B" for at least one of the attributes, then alternative "B" is said to be *dominated* by "A". This concept will be formally defined later in this chapter.

[19] This approach will be explained in the next chapter.

leads to the methods and techniques of multi-objective optimisation, that attempt to explore the efficient set that may exist in up to k dimensions (where k is the number of criteria).

4.4.1 Multi-objective optimisation problem

A general multi-objective optimisation problem includes a set of n decision variables, a set of k objective functions, and a set of m constraints, where objective functions and constraints are functions of the decision variables. Using the notation used in Zitzler (1999) the decision vector is denoted by **x** and **X** denotes the decision space. Similarly, the objective vector is denoted by **y** and **Y** denotes the objective space (see Figure 4-1). The constraints $\mathbf{e}(\mathbf{x}) \leq \mathbf{0}$ determine the set of feasible solutions. Then, the optimisation goal is to:

Maximise: $\mathbf{y} = \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})]$

Subject to: $\mathbf{e}(\mathbf{x}) = [e_1(\mathbf{x}), e_2(\mathbf{x}), \ldots, e_m(\mathbf{x})] \leq \mathbf{0}$

Where: $\mathbf{x} = [x_1, x_2, \ldots, x_n] \in \mathbf{X}$

and $\mathbf{y} = [y_1, y_2, \ldots, y_k] \in \mathbf{Y}$

4.4.1.1 Feasible set

The feasible set $\mathbf{X}_f$ is defined as the set of decision vectors **x** that satisfy the constraints **e**(**x**) as $\mathbf{X}_f = \{\mathbf{x} \in \mathbf{X} | \mathbf{e}(\mathbf{x}) \leq \mathbf{0}\}$ and the feasible region in the objective space, is denoted as $\mathbf{Y}_f = f(\mathbf{X}_f)$.

4.4.1.2 Pareto Dominance:

In single objective optimisation, the feasible set is ordered according to the objective function f(x). For example, for solutions $\mathbf{a}, \mathbf{b} \in \mathbf{X}_f$ either $f(\mathbf{a}) \geq f(\mathbf{b})$ or $f(\mathbf{b}) \geq f(\mathbf{a})$. Therefore, as stated in the previous section, the optimal solution (or solutions) is the one that provides the maximum value of the objective function.

The situation is very different when several objectives are involved. In this case, it is necessary to define the concept of dominance. Dominance implies that if there is an alternative A that is at least as preferred as B for each one of the attributes that are being considered, and if A is strictly preferred to B for at least one of the attributes, then alternative B is said to be dominated by A.

An example of dominance is shown in Figure 4-2 as in Zitzler (1999). This figure represents a solution space with two objectives. The solution represented by point B is better than the solution represented by points C and D as it provides better performance for both objectives $\mathbf{y}_1 = f_1(\mathbf{x})$ and $\mathbf{y}_2 = f_2(\mathbf{x})$. On the other hand the solution represented by point A dominates point B in the same manner. In other words, the light grey rectangle contains the region in the objective space that is dominated by the decision vector represented by B. The dark grey rectangle contains the objective vectors whose corresponding decision vectors dominate the solution represented by B.

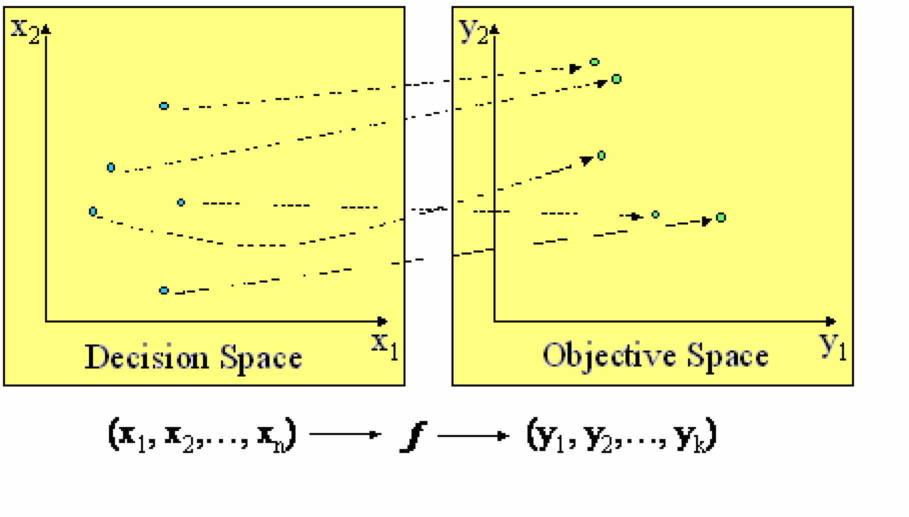

Figure 4-1: Illustration of a general multi-objective optimisation problem. Modified from Zitzler et al. (2004).

4.4.1.3 Pareto Optimality

Having defined the concept of dominance it is possible to introduce the concept of optimality for multiple objective optimisation problems. Referring again to Figure 4-2, the corresponding decision vector **a** of solution **A** is not dominated by any other decision vector. That means that **a** is optimal because it cannot be improved in any objective without causing a degradation in at least one other objective. Such solutions are denoted as Pareto optimal (Zitzler (1999); Graves and Ringuest (2003)).

The red alternatives in Figure 4-2 represent the Pareto-optimal set of solutions. The fact that any of the alternatives in this set is dominated by any other solution makes the DM indifferent to them without any further information about her preferences. Each DM has her own system of preferences that depends on complex behavioural considerations. The value that a DM assigns to an alternative depends on the absolute and relative performance of this alternative on the different relevant attributes. However, the value that a DM gives to any of the non-dominated solutions will always be higher than the value assigned to any of the dominated solutions. Hence, the main idea behind multi-objective optimisation is that regardless of her utility function, a rational DM will not be interested in choosing an alternative that does not belong to the Pareto set.

NOTE: This figure is included on page 34 of the print copy of the thesis held in the University of Adelaide Library.

Figure 4-2: Illustration of Pareto optimality in the objective space. Modified from Zitzler (1999).

4.4.2 Classes of techniques for solving multi-objective optimisation problems

From a practical standpoint a DM needs to choose only one solution from the Pareto set that fits his own system of preferences. As a result, the analysis of a multi-objective problem can be subdivided in two distinct parts, search and preference articulation. The fundamental differences among the different techniques to solve multi-objective optimisation problems come from the ordering in which these two steps are performed.

4.4.2.1 Methods with a priori articulation of preferences

These methods usually avoid the complexities involved in a true multi-objective optimisation problem through the transformation of a multiple objective problem into a single objective one. Hence, these methods do not handle multi-objective optimisation any differently than single objective (Deb (2001)).

Among many others, the most common methods of this type are:

- ε-Constraint method
- Value function method
- Weighted sum method
- Goal Programming methods

In order to set a common ground for discussion in the next chapter, the first two methods from the above list will be briefly explained.

4.4.2.1.1 ε-Constraint method

This method reformulates the multi-objective problem by keeping only one objective and restating the other objectives as goals. Although the concepts of objective and goal are usually used interchangeably, in the multi-objective optimisation jargon an objective indicates a direction in which the

DM must strive to do better. On the other hand, a goal identifies specific levels of achievement. Goals are different from an objective in that is either achieved or not (Keeney and Raiffa (1993)). Consequently, in an optimisation problem, objectives belong to the objective function while goals belong to the set of constraints. As a result, the modified problem is as follows:

Maximise: $f_{\mu}(\mathbf{x}),$

Subject to: $f_k(\mathbf{x}) \geq \varepsilon_k,$ $\quad k = 1,2,\ldots,K$ and $k \neq \mu;$

$e_m(\mathbf{x}) \leq 0$ $\quad m = 1,2,\ldots,M;$

Where: $x_n \in \mathbf{x}$ $\quad n = 1,2,\ldots,N;$

In this formulation, the parameter ε_m represents a lower bound[20] for the value of f_m. Let say that in a bi-objective problem the DM decides to keep f_1 as an objective and treat f_2 as a constraint $f_1(\mathbf{x}) \geq \varepsilon_1$ (Figure 4-3). The constraint represented by the red line divides the objective space in two. All the solutions to the left of the red line now become unfeasible and hence, the solution "E" becomes the optimal solution.

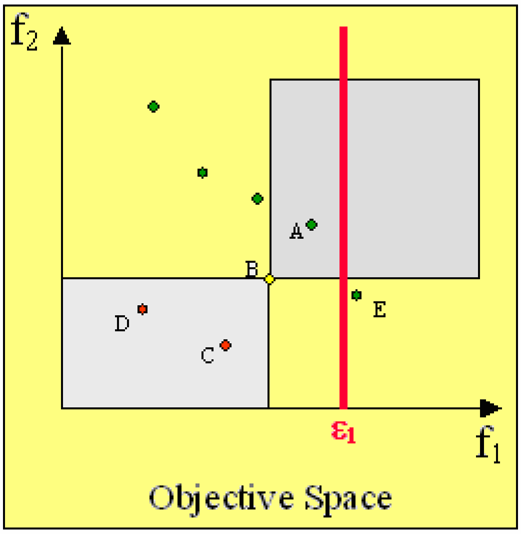

Figure 4-3: The ε– constraint method.

According to Deb (2001), the main advantages and disadvantages of the ε– constraint method are as follows:

[20] For a minimisation problem, this parameter represents an upper bound instead.

Advantages:

- This method guarantees that if an optimal solution is found whilst accounting for a given set of constraints, this solution is Pareto optimal.
- Different optimal solutions can be found using different ε_m values.

Disadvantages:

- The solution of a ε– constraint mathematical program strongly depends on the chosen ε– vector of constraints. Hence, finding a feasible solution can become a difficult task as the number of objectives increase.

4.4.2.1.2 Utility Function Method

To approach an optimisation problem with M objectives, the Utility function method requires the DM to provide a multi-attribute utility function that relates all M objectives Rosenthal (1985). Deb (2001) states that assuming such a function exists for the entire feasible search space the multiple objective optimisation problem can be reduced to a single objective optimisation problem as follows:

Maximise: $\mathrm{U}(\mathbf{f}(\mathbf{x}))$

Subject to: $e_m(\mathbf{x}) \leq 0 \qquad m = 1,2,\ldots,M;$

Where: $x_n \in \mathbf{X} \qquad n = 1,2,\ldots,N;$

Figure 4-4 depicts the contours of a value function in a bi-objective space. According to the figure it is possible to note that although solutions C and B are Pareto optimal, U(**f**(C))> U(**f**(B)) and hence solution C is preferred to solution B. Therefore, with this method it is not necessary to identify the complete set of Pareto optimal solutions. When a multi-attribute utility function is available this procedure will find the solution that is optimal in terms of the preferences of the DMs in a single step.

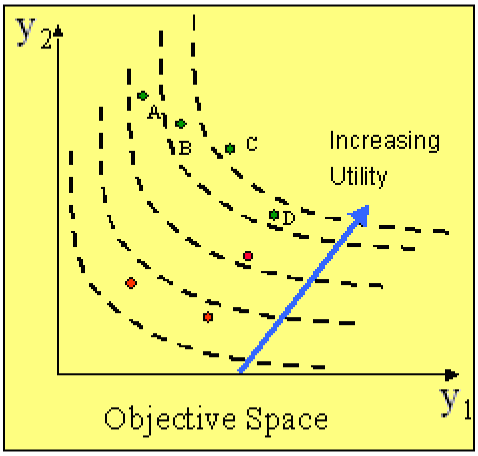

Figure 4-4 Contours of the multi-attribute utility function (Modified from Deb (2001))

Rosenthal (1985) argues that this method is rarely used for problems with a large set of feasible solutions, and that its application has been mostly restricted to problems where the feasible region is small enough to enumerate totally.

Advantages:

- If a utility function is available, this method is the ideal one as it finds in one step the solution that is optimal in accordance to the DMs preferences.

Disadvantages:

- As it was shown in the previous chapter, finding a multi-attribute utility function that truly represents the preference system of the DM and the tradeoffs the she is willing to make is not an easy task at all. Keeney and Raiffa (1993) present a vast compendium of techniques to assess preferences from DMs in order to construct a multi-attribute utility function. However, Deb (2001) argues that this approach there is always danger of using an oversimplified multi-attribute function.

4.4.2.2 Methods with a posteriori articulation of preferences

Methods with posterior articulation of preferences begin by generating a representative set of non-dominated solutions. Once this representative set is calculated it is necessary to solicit trade-off information from the DM that is used to select a preferred solution from this set (Graves and Ringuest (2003)). The quality of a multi-objective optimisation method with posterior articulation of preferences mostly depends on two issues (Deb (2001)), (Zitzler et al. (2004)):

- Finding a set of solutions as close as possible to the Pareto optimal front.

- Finding a set of solutions that are sparsely spaced across the range of the Pareto-optimal region.

There are two main approaches to generate a set of Pareto-optimal solutions. The first approach consists in performing several runs of methods with “a priori” articulation of preferences where in each run the “preferences” (i.e., weights in the multi-attribute utility function, goals in the ε -constraint method) of the DM are systematically altered. These methods are also called “exact” multi-objective methods (Zitzler et al. (2004)) since the solutions provided by these methods belong to the Pareto optimal set as each one of the single optimisation provides the actual optimum value. The main disadvantage of these methods is that they are not guaranteed to provide solutions that are sparsely spaced in situations where the shape of the Pareto surface is discontinuous or concave (Coello (2001)). The main reason for this is that systematic changes in the preferences of the DMs, for example via a multi-attribute utility function, do not necessarily provide a uniform spread of samples in the objective space. On the other hand, finding feasible solutions through systematic changes in constraints set using the ε -constraint method may be a difficult task as a result of the trade-offs among the attributes under study.

The second approach is also called “search” approach. This approach consists in generating solutions, testing them in the objective set of functions and keeping the ones that are non-dominated by previous solutions. According to Zitzler et al. (2004), search methods algorithms usually consist of three parts, solution generator, a working memory that contains the currently generated solution and a selection module that compares new solutions with old ones and discard dominated ones while keeping the non-dominated ones. Figure 4-5 shows that as new solutions are generated it is possible to generate an approximate Pareto set that eventually will converge towards the real Pareto set.

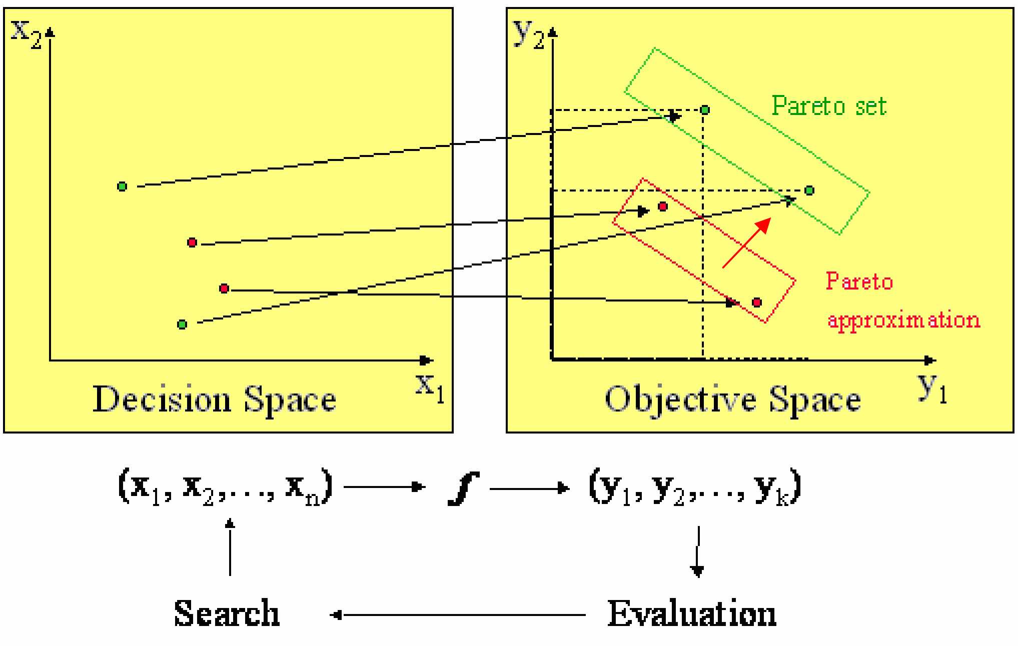

Figure 4-5: Approximation of the Pareto set through iterative evaluation and search. Source Zitzler et al. (2004).

The simplest approach to a “search” based method is to use a random solution generator. The larger the number of solutions generated the closer the approximate set will be to the true Pareto set. However, the computational cost of doing this is extremely high and hence a number of meta-heuristic methods (e.g., evolutionary algorithms, tabu search, simulated annealing) have been developed to generate solutions in such a way that it is possible to calculate a good approximation of the true Pareto with affordable computing power. Evolutionary algorithms are particularly suitable to solve multi-objective optimisation problems because they deal simultaneously with a set of possible solutions and this allows finding several potential members of the Pareto optimal set in a single run of the algorithm (Coello (2001)). Moreover, the evolutionary approach is less susceptible to the shape or continuity of the Pareto set than the “exact” approaches.

However, as it was shown in Figure 4-5 the main disadvantage of these methods is that, although they are superior to the “exact” methods regarding the preservation of the efficient set, they do not always provide an “exact” solution but rather an approximation of the “true” Pareto set.

4.4.3 Stochastic optimisation

In the multi-objective methods problem previously shown the objective function coefficients is a set of scalar valued functions (Rosenthal (1985)). This means that these methods assume that it is possible to know with certainty the future performance of a given solution in the objective space. However, in many cases the DM not only wishes to optimise several objectives at the same time, but moreover, there is uncertainty in the future performance of potential solutions. When the objective function coefficients are random, the resulting problem is denominated stochastic multi-objective optimisation (Caballero et al. (2001)). This kind of programs can be defined as follows:

Maximise: $[z_1(\mathbf{x},\omega),\ldots,z_k(\mathbf{x},\omega),\ldots,z_K(\mathbf{x},\omega)]$

Subject to: $e_m(\mathbf{x}) \le b_m, m = 1,\ldots,M$

$$\mathbf{0} \le \mathbf{x} \le \mathbf{1}$$

Where **x** is a vector of decision variables of dimension nx1. For k=1,…,K, z_k(**x**, ω) is the kth objective and ω represents the stochastic effect in the objectives. For i=1,…,M, e_m(**x**) denotes a linear constraint in the decision vector **x**, and b_m is the right hand side scalar for the mth constraint.

4.4.3.1 Multi-objective stochastic optimisation

As in the multiple objective problem previously defined, it is often the case that the objectives of the stochastic multi-objective optimisation conflict with each other. Therefore, the problem is not to find a unique “optimal” solution **x,** but to identify a set of solutions that are not dominated in a stochastic sense. In other words, because each one of the objectives for a given solution is a probability density distribution, it is necessary to compare different statistics of these distributions.

Caballero et al. (2001) state that different concepts of stochastic dominance can be found in the literature. However, most of the cases seek to optimise either a central tendency measure (e.g., expected value) and a dispersion metric (e.g., standard deviation) or a central tendency measure and a given percentile (e.g., P10, P90)[21]. In this manner, under uncertainty, the objectives of the optimisation problem are not set over the performance attributes themselves but over statistics extracted from their PDFs.

To illustrate the concept of dominance using of statistics extracted from PDFs, the stochastic dominance criteria used in Medaglia (2003) will be described. This approach defines the concept of stochastic dominance in terms of expected value and the probability of achieving a target value specified by the DM as follows.

Let **x** and **x'** be two feasible decision variable vectors in a stochastic multi-objective problem (maximisation). Solution **x** dominates solution **x'** if and only if for all k, $E[z_k(\mathbf{x})] \geq E[z_k(\mathbf{x}')]$ and $P\{z_k(\mathbf{x}) \geq T_k\} \geq P\{z_k(\mathbf{x}') \geq T_k\}$, and there exists at least one k, such that $E[z_k(\mathbf{x})] > E[z_k(\mathbf{x}')]$ or $P\{z_k(\mathbf{x}) \geq T_k\} > P\{z_k(\mathbf{x}') \geq T_k\}$.

Where $E[z_k(\mathbf{x})]$ is the expected value of the kth objective ($z_k(\mathbf{x})$) and $P\{z_k(\mathbf{x}) \geq T_k\}$ is the probability that $z_k(\mathbf{x})$ is greater than or equal to a target value T_k specified by the DM.

Medaglia (2003) also states that if the DM is not concerned with the probability of achieving certain target but rather prefers to maximise a given percentile of the PDFs of the attributes it is convenient to rewrite the stochastic dominance statement in the following manner. If $P\{z_k(\mathbf{x}) \geq T_k\} = F_k$ and $P\{z_k(\mathbf{x}') \geq T_k'\} = F_k$, with F_k specified by the DM, then solution **x** dominates solution **x'** if and only if the following two conditions hold:

For all k, $E[z_k(\mathbf{x})] \geq E[z_k(\mathbf{x}')]$ and $T_k \geq T_k'$, and there exists at least one k, such that $E[z_k(\mathbf{x})] > E[z_k(\mathbf{x}')]$ or $T_k > T_k'$.

However, since it is not possible to parameterise the probabilities of achieving a certain target value or the percentiles of a distribution through an equation, Medaglia (2003) proposes the use of a simulation-optimisation approach to solve this problem. The simulation-optimisation approach is not exclusive for

[21] This notation is commonly used in the oil & gas literature to denote the 10^{th} percentile and the 90^{th} percentile respectively.

multi-objective problems, and it generally uses a combination of Monte Carlo simulation with meta-heuristics search methods. The main advantages of this methodology are:

- It respects the dependencies among the system random variables.
- It allows optimising any statistic from an attribute modelled as a random variable (stochastic objectives).
- It allows setting constraints over any statistic from any random variable.

A multi-objective simulation-optimisation approach with posterior articulation of preferences has the previous advantages plus the capability to produce approximate Pareto sets in terms of objectives set over certain statistics of the attributes under study.

4.4.3.1.1 Parameter Space Investigation Method (PSI)

The parameter space investigation method was originally described by Sobol (1992). This method was originally conceived to solve for deterministic multi-objective nonlinear problems. However, Graves and Ringuest (2003) propose a modified version of this approach for problems with stochastic objectives and a deterministic set of constraints.

These authors state that a stochastic PSI method would proceed as follows:

- A preset number of trial points are randomly generated based on the bounds of the decision variables.
- If a trial point is not feasible it is discarded.
- If the trial point is feasible, the relevant statistics over the set of performance attributes is estimated through Monte Carlo simulation.
- If the solution of a trial set is dominated by any previous solution it is discarded. Any remaining solutions are presented to the DM.

This simple methodology generates a sample of non-dominated solutions for a stochastic multi-objective problem. However, it is important to note that the set of solutions is non-dominated for the particular sample produced by the trial points and, therefore, it is not guaranteed to be a good approximation of the Pareto set.

4.5 Multi-objective genetic optimizer with linear constraints (MOGOL).

Medaglia (2003), presents an evolutionary algorithm to solve for multi-objective linearly constrained problems under uncertainty (MOGOL). This algorithm considerably improves the solution time and the quality of the solutions, in terms of being more evenly spaced and closer to the “true” Pareto optimal set, if compared to the stochastic PSI method proposed by Graves and Ringuest (2003). Note that unless another author is mentioned all the information presented in this section is extracted from Medaglia (2003).

MOGOL was initially developed to help DMs in the selection of generic research and development projects under uncertain conditions and multiple objectives. Since research and development projects are usually linearly constrained by a budgetary constraint, MOGOL is designed to take advantage of this characteristic to increase the solution speed and the accuracy of the approximate Pareto set. Hence, its application is narrowed to problems with linear deterministic constraints (i.e. the feasible region needs to be convex). The reason for this is that, as opposed to the PSI method, the MOGOL directly searches the feasible region.

Figure 4-6 shows a hypothetical linearly constrained feasible region marked by the yellow area, where the decision space has two decision variables (x_1, x_2). To generate new individuals MOGOL firstly calculates extreme points of the feasible region, denoted in Figure 4-6 by the black dots.

Medaglia (2003), states that the calculation of these extreme points is fundamental for the MOGOL algorithm. This statement is based in the fact that, as shown in Figure 4-6, feasible solutions $\mathbf{x_a}$ and $\mathbf{x_b}$ can be generated as convex combinations of the extreme points $\mathbf{x_1}$ and $\mathbf{x_4}$, and $\mathbf{x_2}$ and $\mathbf{x_5}$ respectively. Where $\mathbf{x_a} = \alpha_a \mathbf{x}_1 + (1-\alpha_a)\mathbf{x}_4$ with $0 \le \alpha_a \le 1$ and $\mathbf{x}_b = \alpha_b \mathbf{x}_2 + (1-\alpha_b)\mathbf{x}_5$ with $0 \le \alpha_b \le 1$. Similarly, it is possible to generate any other point (e.g., $\mathbf{x_c}$) by successive convex combinations of previous solutions that can be traced back to the extreme points.

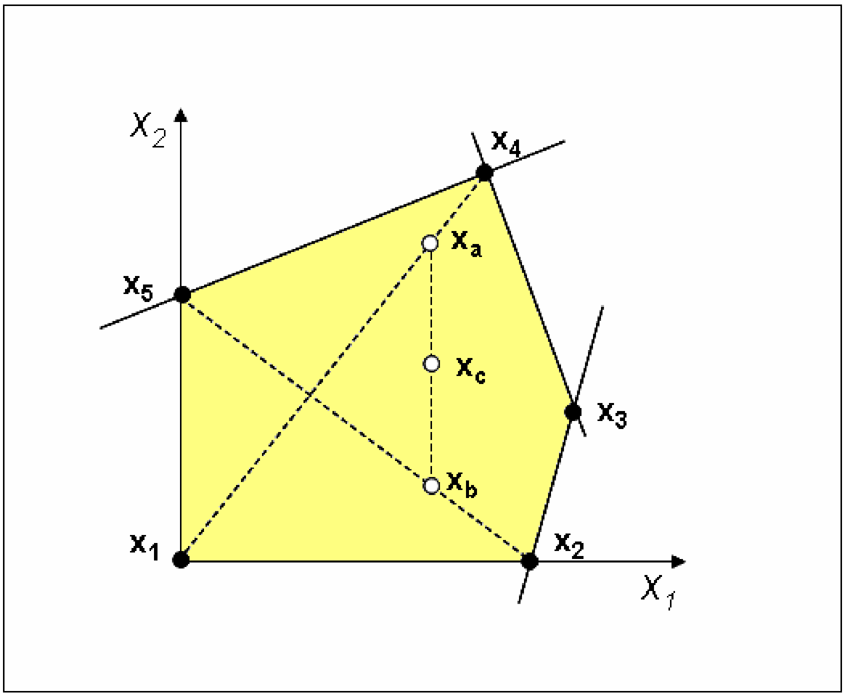

Figure 4-6 Feasible region. Modified from Medaglia (2003)

A detailed explanation of the MOGOL algorithm described in Medaglia 2003 goes beyond the scope of this thesis. However, since a posterior version of the MOGOL algorithm to the one here presented is used as the search engine for the E&P project portfolio model proposed in this thesis, it is convenient to describe how the algorithm produces an approximate efficient frontier, at least from a high level perspective.

Firstly, the user must enter the set of inputs that define the characteristics of the projects and the constraints of the problem. In this manner the user must enter the set of constraints of the problem (required to be linear), the PDFs of the performance attributes (defined as triangular distributions) for each of the project proposals and the objective functions to be optimised.

Secondly, the user must also define a series of parameters that will impact the quality of the solution and the solution time of the algorithm. These inputs are the number of Monte Carlo independent replications desired for each simulation, the number of generations of solutions to be calculated (N_{max}), the size of the population (P(t)), a crossover probability (P_c) and a mutation probability (P_m).

Once this set of inputs and parameters is set it is possible to run the algorithm. Initially the algorithm calculates the extreme points of the feasible solution as explained above and produces an initial population (P(1)) of solutions where each solution is defined by a participation vector. Each participation vector represents a feasible portfolio. Then a built-in Monte Carlo simulator calculates full PDFs of the objective functions and extracts the statistics of interest in a batch basis for each one of the portfolios of the population.

Then the algorithm recombines the solutions obtained in P(1) by crossover. This procedure seeks to combine the "genetic" material of two "parent" portfolios to obtain a "child" solution that resembles them. Every solution from P(1) has a probability p_c (defined by the user) of being selected. The selected solutions are then paired to produce a child in the following manner. If $\mathbf{x}_a$ and $\mathbf{x}_b$ are parent solutions selected for the crossover, the "child" $\mathbf{x}_c$ will be defined as $\mathbf{x}_c = \alpha \mathbf{x}_a + (1-\alpha)\mathbf{x}_b$, where α is a random number selected from a uniform distribution over the range [0,1]. The new child $\mathbf{x}_c$ forms part of $C_c(1)$, a population of new solutions generated from the crossover of the selected solutions from P(1).

In addition to the new children generated by crossover, the algorithm also injects a percentage p_m (defined by the user) of new individuals. These new solutions form $C_m(1)$, a population of new solutions generated from this mutation process.

After $C_c(1)$ and $C_m(1)$ are calculated these two populations are combined to form the total children population C(1). The solutions contained in C(1) are then evaluated using Monte Carlo to extract the statistics of relevance according to second stochastic dominance criteria proposed in section 4.4.3.1.

Figure 4-7 is a flow chart that shows how the MOGOL evaluates a given solution, the chart assumes that the DM is interested in optimising the expected value and the P90 of the attributes under study.

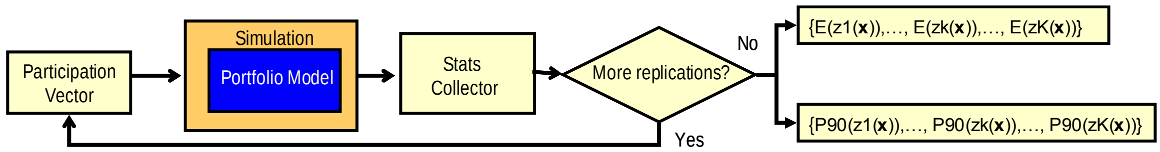

Figure 4-7 Flow chart describing the evaluation of possible solutions by MOGOL

Once C(1) has been evaluated, this population is combined with P(1) and the resulting population (E(1)) sorts the individuals according to various layers of non-domination based on the relevant statistics extracted from the PDFs of the objective functions. This procedure firstly calculates the non-dominated solutions of E(1), then the algorithm sets this first layer of non-dominated solutions (F_1) aside and calculates a new set of non-dominated solutions (F_2) with the remaining solutions. Similarly, it puts F_2 aside and calculates F_3 and this process continues until all the solutions contained in E(1) are classified into one frontier.

Once all the solutions that form the initial population (P(1)) and the children population C(1) have been classified into one frontier, the algorithm selects new individuals for the next generation. The algorithm does this by ranking the solutions starting with the solutions belonging to the best non-dominated frontiers until the maximum number of individuals is reached. In other words, all the individuals from F_1 are selected, then all the individuals from F_2 selected and so on until the maximum number of individuals for the next generation (P(2)) is reached. Since it may not be possible to select all the solutions in the last frontier when P(2) is reached, the algorithm uses the crowding distance sorting method proposed by Deb (2002). The crowding distance method seeks to select the solutions that are as sparsely spaced as possible across the Pareto front by literally avoiding solutions clustered around a specific point in the non-dominated front (Coello (2001)). The main reason to do this is that these points are quasi-redundant in terms of information to the DM.

The fact that the best individuals from the best fronts are always selected, they always produce more children solutions than the rest of the population. This feature of the MOGOL algorithm and the crowding distance concept are elements borrowed from the Non-dominated Sorting Genetic Algorithm II (NSGA-II) described in Deb (2002).

The MOGOL then starts the whole procedure again using the new population (P(2)) and so on until the maximum number of generations specified by the user is reached. The whole procedure of the algorithm can be summarised then in the following manner, where $F = F_1 \cup F_2 \cup F_3 \cup \cdots$ and N_{max} is the number of generations defined by the user:

$t \leftarrow 1$

Initialize population P(1)

Evaluate population P(1)

While t<=N_{max}

Recombine P(t) to generate C_c(t)

Alter P(t) to generate C_m(t)

$$C(t) = C_c(t) \cup C_m(t)$$

Evaluate children population $C(t)$

$$E(t) = P(t) \cup C(t)$$

Sort E(t) by stochastic non-domination to generate F

Select P(t) from F

Where at the end the approximate Pareto set will be equal to P(N_{max}).

4.5.1 Discussion of the MOGOL algorithm

The MOGOL algorithm proposed in Medaglia (2003) provides a robust way to produce Pareto sets of generic research and development projects with multiple objectives under uncertain conditions. However, the first version of the algorithm Medaglia et al. (2004) described in the previous section lacked the ability to account for the inter-project and intra-project correlations that are present in E&P projects. This thesis led to the development a second version of the MOGOL algorithm that does account for these correlations as explained in the following chapters.

4.6 Summing up

This chapter described the main characteristics of a multi-objective optimisation problem. The fact that it is not possible to obtain an optimal solution in a multi-objective sense without the preferences of the DM was addressed. Additionally, the chapter highlighted the fact that multi-objective problems can be broadly classified depending on whether the preferences of the DM are articulated before or after the optimisation is performed.

The chapter also showed that most of the classic multi-objective methods with posterior articulation of preferences are usually based on several runs of methods with a priori articulation of preferences. This practice produces solutions that are Pareto optimal on an individual basis but that are not necessarily evenly spaced across the full range of the Pareto optimal surface. Additionally, it was shown that these methods fail to handle goals and/or objectives over statistics that cannot be described through an equation (i.e., percentiles, probabilities of achieving a certain target).

On the other hand, evolutionary algorithms provide solutions that are more evenly spaced over the Pareto optimal surface. Additionally, it was described how these algorithms can be used to set goals and objectives over any statistic when combined with Monte Carlo simulation through the simulation-optimisation approach.

It is important to highlight that this chapter presented the main difference between goal and objective where, in this context, an objective indicates a direction in which the DM must strive to do better whereas a goal identifies specific levels of achievement.

Lastly, the chapter described the multi-objective optimisation algorithm with linear constraints (MOGOL) proposed in Medaglia (2003).

5 OIL AND GAS PORTFOLIO OPTIMISATION

5.1 Introduction

Firstly, this chapter defines how to aggregate the variables of individual projects in order to measure the performance of a potential portfolio of E&P projects. Secondly, this chapter draws in the oil and gas decision and risk analysis literature to review the classical and state of the art methodologies to optimise the selection of E&P projects. Lastly, the advantages and disadvantages of these methodologies are discussed.

5.2 Aggregating projects: deterministic and stochastic approaches

After the asset valuation stage has been completed and the relevant attributes and correlations have been quantified, the DMs need to search for the optimal investment alternative accounting for the current objectives and/or goals and available resources of the company. However, the optimisation problem has to be set over a model that aggregates the performance attributes of the project proposals so that the performance of the portfolio in those attributes can be quantified.

The working interest[22] represents the percentage owned by the company in a specific project. Most[23] of the performance attribute of a portfolio (Z_{Port}) can be expressed as a linear combination of the same attribute for each asset (z_i) proportioned by its working interest (x_i). In this manner the performance of a portfolio in the performance attribute Z can be expressed as:

$$Z_{\text{Port}}(\mathbf{x}) = \sum_{i=1}^{N} x_i z_i$$

Thus, in a deterministic E&P portfolio optimisation problem with N assets, where **Z** is a performance attribute to be optimised, the objective function set over that attribute can be defined as follows:

$$\textbf{Maximise: } Z_{\text{Port}}(\mathbf{x}) = \sum_{i=1}^{N} x_i z_i$$

Alternatively, if $\mathbf{Z}_{Port}$ is required to reach a goal T it can be expressed in the set of constraints as:

$$Z_{\text{Port}}(\mathbf{x}) = \sum_{i=1}^{N} x_i z_i \geq T$$

[22] Working interest and participation level are used indistinctively.

[23] Efficiency measurements as the net present value over investment (NPV/Inv) ratio cannot be calculated as a linear combination of the performance of each of the projects in the portfolio for this attribute times its working interest. To calculate an efficiency measure of a portfolio it is necessary to firstly calculate the NPV and the total investment (Inv) of the portfolio and then calculate the ratio of these two attributes.

If projects are characterised stochastically and DMs are interested in optimising statistics of the PDFs of the performance attributes, then it is necessary to restate the objective function(s) related to each attribute. If for example the corporate DMs are interested in maximising the expected value and the 10th percentile of attribute Z_1 and maximising the expected value of attribute Z_2 while minimising the standard deviation of attribute Z_2 then the objective functions of the multi-objective optimisation would be as follows:

Maximise: $E[\mathrm{Z}_{1\mathrm{Port}}(\mathbf{x})] = E\left[\sum_{i=1}^{N} x_i z_{1_i}\right]$

Maximise: $P10[\mathrm{Z}_{1\mathrm{Port}}(\mathbf{x})] = P10\left[\sum_{i=1}^{N} x_i z_{1_i}\right]$

Maximise: $E[\mathrm{Z}_{2\mathrm{Port}}(\mathbf{x})] = E\left[\sum_{i=1}^{N} x_i z_{2_i}\right]$

Minimise: $SD[\mathrm{Z}_{2\mathrm{Port}}(\mathbf{x})] = SD\left[\sum_{i=1}^{N} x_i z_{2_i}\right]$

Similarly, if the DMs are interested in investing in a portfolio with at least 90% probability of achieving a positive value for attribute Z1, then the following statement should appear in the constraints of the optimisation statement:

$$\mathrm{P}[\mathrm{Z}_{1\mathrm{Port}}(\mathbf{x}) > 0] = P\left[\sum_{i=1}^{N} x_i z_{1_i} > 0\right] \geq 0.9$$

It is possible to note that in both the stochastic and the deterministic cases the portfolio attributes are a function of the set of decision variables that conform **x.** Each solution of **x** is a portfolio that describes a unique investment strategy for the available opportunities. Corporate DMs may define a given investment strategy in terms of two different types of decision variables: working interest and timing. If the DMs are just concerned about finding the optimal working interest, they may model a decision vector as follows:

$$\mathbf{x}^{\mathbf{T}} = [x_1, x_{2,} \ldots, x_N]$$

$$0 \leq x_i \leq 1$$

Where X is a participation vector and x_i represents the participation level on the i^{th} project. Usually, x_i is modelled as a continuous variable that can take any value between zero and one[24].

Yet, most companies will try to avoid strong fluctuations in the yearly future profiles of their performance attributes. For these reason, corporate DMs might not only be interested in optimising the working interest but also the timing to start the projects. To do so, it is necessary to restate the decision vector as a 2xN matrix.

$$\mathbf{x}^{\mathbf{T}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ t_1 & t_2 & \cdots & t_N \end{bmatrix}$$

$$0 \leq x_i \leq 1$$

$$0 \leq t_i \leq k_i$$

Where the first row is similar to the participation vector described previously and the second row of the matrix specifies the timing of the i^{th} project. The variable t_i is discrete and can vary from zero (start the project now) to a given number of years into the future (k_i). Fichter (2000) states that in practice, a project may have a typical starting time span between one and five years (k_i~5 years).

5.3 E&P portfolio optimisation methods most commonly found in the literature

In this section the capital rationing approach and the mean-variance approach will be described. To aid with the illustration of these methods, a hypothetical simple portfolio optimisation problem will be used. The problem assumes that a company wishes to optimise an eight project portfolio with a budget of US$400 million. This company has access to a set of eight projects and is mainly concerned about the future performance of their asset portfolio in terms of two metrics: net present value (NPV) and total reserves added (i.e., total cumulative production). The projects have been evaluated stochastically in terms of both attributes using Monte Carlo simulation.

The possible NPV and cumulative production outcomes are chosen to be triangular distributions with minimums, modes and maximums as represented in table 5-1. This table also shows the cost of the

[24] However, the participation level in a particular project may be constrained because other company has already taken a portion of it. Another possible constraints of the decision variables is the fact that contractual agreements of the country and/or the corporation promoting the project might require a minimum level of working interest to participate in the venture.

projects (assumed to be deterministic). Note that the eight projects require a capital of US$725 millions to fund fully.

		NPV ($MM)					Cumulative Production (MMBOE)				
	Cost ($MM)	Min	Mode	Max	Mean	SD	Min	Mode	Max	Mean	SD
P1	100	-10	25	60	25	14.29	5	12	25	14	4.14
P2	70	-30	20	85	25	23.54	2	5	15	7.33	2.78
P3	80	-40	10	90	20	26.77	2	10	20	10.67	3.68
P4	105	-20	15	40	11.67	12.3	5	15	25	15	4.08
P5	85	-5	20	45	20	10.21	5	10	20	11.67	3.12
P6	60	-15	5	20	3.33	7.17	1	3	10	4.67	1.93
P7	65	-5	10	25	10	6.12	1	3	10	4.67	1.93
P8	160	-25	-5	60	10	18.14	15	25	35	25	4.08

Table 5-1 Costs and distribution parameters of the NPV and cumulative production metrics

With a capital budget of US$400 million, the DMs of the company need to choose a portfolio that delivers the best risk and return combination for both metrics in accordance to their system of preferences. The following assumptions will be made:

- The company can participate in a project at any level. As a result, working interest can take any value between 0 and 1.
- Transaction costs are assumed to be included in the overall investment for each project.
- DMs are just concerned about optimising working interest. Consequently, timing is ignored.
- All projects are independent. All inter-project correlations are set equal to zero.

5.3.1 Capital rationing approach

Several authors (Orman and Duggan (1998); Ball and Savage (1999a); Ball and Savage (1999b); Brashear et al. (2000); Bratvold et al. (2003); Campbell et al. (2003)) have highlighted the fact that even at the present time the most common approach to project selection and capital allocation is the capital rationing approach, most commonly named as the "rank and cut" method.

As stated before, DMs do not have the required capital to fully develop all the available projects. The "rank and cut method" addresses this budgeting constraint by evaluating projects on individual merits. In this method, projects are ranked on a profitability index such as net present value to investment (NPV/Inv), and funded until the budget is committed. As a consequence of the above, all projects receive a 100% working interest with the possible exception of the last project selected, which may receive partial funding.

5.3.1.1 Capital rationing optimisation problem

Maximise: $NPV_{\mathrm{Port}}(\mathbf{x}) = \sum_{i=1}^{n} x_i NPV_i$

Subject to: $Cost_{\mathrm{Port}}(\mathbf{x}) = \sum_{i=1}^{n} x_i Cost_i = Budget_{Port}$ and $0 \le x_i \le 1$

This procedure maximises the return on invested capital, but if and only if the future project outcomes are as predicted in the analysis that guided the ranking or, if the project outcomes are not as predicted but their relative ranking remains constant. Consequently, the "rank and cut" method overlooks the fact that the future performance of oil and gas assets is uncertain and, as a result, there are risks associated to each investment. Additionally, this method overlooks the existence of correlations between projects and the fact that E&P firms may also be measuring their performance against more than one metric for several years into the future.

Note that the companies performing the rank and cut method usually do not derive the expected values of the NPV from Monte Carlo simulation on stochastic valuation models. In contrast, they valuate their projects with deterministic methods that use single estimates of the expected values for each of the parameters used in the valuation process and as a result, the output is a "best estimate" of the expected value of the NPV.

However, if projects are evaluated stochastically and ranked according to the expected value of their NPV/Inv ratio (E(NPV/Inv)), then this method maximises the utility of a risk neutral investor if projects are assumed to be independent.

5.3.1.2 Example

In order to solve the hypothetical portfolio optimisation problem presented at the beginning of this section with the capital rationing approach it is necessary to state the problem as follows:

Maximise: $NPV_{\mathrm{Port}}(\mathbf{x}) = \sum_{i=1}^{8} x_i NPV_i$

Subject to: $Cost_{\mathrm{Port}}(\mathbf{x}) = \sum_{i=1}^{8} x_i Cost_i = 400 MM\$US$ and $0 \le x_i \le 1$

Table 5-2 states that using the set of projects described above it is possible to note that when the projects are ranked according to their level of capital efficiency, the resulting portfolio is $X_{rank\&cut}$ = [1, 1, 1, 0, 1, 0, 1,0]. This vector means that according to this method, projects 1, 2, 3, 5 and 7 should receive full funding while projects 4, 8 and 6 should not receive any funding at all. Since the total cost of the projects selected equals the total available budget any of the selected projects receives partial funding. This portfolio is supposed to deliver a NPV of US$ 100MM. Additionally, as a by-product of the maximisation of the NPV the portfolio delivers 48.4MM bbls of reserves.

Project	NPV	NPV/Inv	Reserves	Cost
	$MM		MMbbl	$MM
P2	25.0	0.4	7.3	70.0 ✓
P1	25.0	0.3	14.0	100.0 ✓
P3	20.0	0.3	10.7	80.0 ✓
P5	20.0	0.2	11.7	85.0 ✓
P7	10.0	0.2	4.7	65.0 ✓
P4	11.7	0.1	15.0	105.0 ✗
P8	10.0	0.1	25.0	160.0 ✗
P6	3.3	0.1	4.7	60.0 ✗

Table 5-2 Results of the capital rationing method

Advantages:

- Maximises the return on invested capital, but if and only if the future project outcomes are as predicted, or if the projects do not perform as predicted by their relative ranking remains constant.
- If the projects are evaluated stochastically and the expected value of the NPV/Inv is used in the ranking, it maximises the utility of a risk neutral investor.
- It is an appropriate tool for a first screening of the project proposals.
- Its calculation is strait forward does not require any optimisation software.

Disadvantages:

- It ignores uncertainties and associated risks in the attributes of the project proposals.
- It ignores the inter-project correlations.
- It is not suitable for risk-averse investors or investors with a multi-attribute utility function.
- It does not account for the “time to invest” decision variable.

5.3.2 Mean-variance approach

Nobel laureate Harry Markowitz published this method in 1952 (Markowitz (1952)). This article and subsequent publications demonstrated how investors of financial securities could minimize the risk while maximising the returns of their investment portfolio. Markowitz stated that exposure to risk could be minimized by understanding the relationship among different stocks. He pointed out that the risk reducing effects of diversification are reduced if multiple investments are positively correlated but amplified if the investments are negatively correlated. Moreover, Markowitz stated that there is a combination of securities that maximizes the return for each level of risk.

The mean-variance method replaced the single-objective approach to investment, focusing solely on an expected measure of return or capital efficiency, by a two-objective approach. The two-dimensional approach characterizes a portfolio in terms of expected return and a measure of dispersion.

Several oil and gas authors (Hightower and David (1991); Orman and Duggan (1998); Ball and Savage (1999a); Brashear et al. (2000); Simpson (2002); Bratvold et al. (2003)) show the benefits of adapting the mean-variance method to the E&P resource allocation problem. These authors state that the main difference between financial stock portfolios and E&P portfolios is that stock portfolios are concerned only with the proportions of various assets held, regardless of the size of the budget. As a consequence, the return of stock portfolios is measured as an annualised average percentage return that is independent of the budget of the investor. In contrast, E&P portfolios usually consist of projects where the company takes a significant portion, and arbitrary fractional investments are not available. For these reasons the return of E&P capital projects is usually measured directly in terms of the NPV and is constrained by the capital available. Note that the set of equations presented next are based on Bratvold et al. (2003).

If NPV_{ij} denotes the jth possible outcome for the NPV attribute on the ith project and P_{ij} the probability of the jth return on the ith project, then the expected NPV of the ith project is given by:

$$E(NPVi) = \sum_{j=1}^{M} P_{ij} NPV_{ij}$$

The expected net present value (E(NPV)) of a portfolio of E&P projects is simply a weighted average of the expected NPV of the individual project proposals proportioned by the working interest in that project. In this manner, the expected return of a portfolio of N projects can be expressed in the following form:

$$E(NPV_{Port}) = \mathbf{X}^{\mathbf{T}}\mathbf{E}(\mathbf{NPV}) = \sum_{i=1}^{N} x_i E(NPV_i)$$

Where the weight (x_i) is the participation level on the i^{th} project is the vector previously defined in equation 4 and the vector of project returns is denoted by:

$$E(\mathbf{NPV}) = \begin{bmatrix} E(NPV_1) \\ E(NPV_2) \\ E(NPV_3) \\ \vdots \\ E(NPV_N) \end{bmatrix}$$

On the other hand, the variance of a portfolio (σ_{Port}^2) is a function of the variances of the individual projects and the covariance between the projects and can be expressed as:

$$\sigma_{Port}^{\ 2} = \mathbf{X^T S X} = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \right]$$

Where **S** is the variance-covariance matrix defined as:

$$\mathbf{S} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3N} \\ \vdots & & & & \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN} \end{pmatrix}$$

And σ_i^2 is the variance of the NPV of the ith project defined as:

$$\sigma_i^{\ 2} = \sum_{j=1}^{M} \left[P_{ij} (NPV_{ij} - E(NPV_i)^2 \right]$$

The standard deviation is defined as the squared root of the variance.

$$\sigma_p = \sqrt{\mathbf{X^T S X}} = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \right]^{1/2}$$

The covariance between two projects can be expressed as a function of the variance of each project and its correlation coefficient as follows:

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$$

And hence the standard deviation of the portfolio can be re-expressed as:

$$\sigma_p = \sqrt{\mathbf{X^T S X}} = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_i \sigma_j \rho_{ij} \right]^{1/2}$$

With the definition of these parameters it is possible to represent graphically all conceivable combinations of projects in an E(NPV) vs. standard deviation of the NPV (SD(NPV)) space. Since investors prefer to maximise returns whilst minimising risk, it is important to identify a set of portfolios that offer a bigger return for the same risk, or offer a lower risk for the same return. This set of portfolios is bi-objective Pareto set commonly called efficient frontier in the finance jargon.

5.3.2.1 Mean Variance Optimisation

As stated before, the mean-variance framework to optimise a portfolio of investments has two objectives (maximise return and minimise risk[25]). Since it was shown that it is possible to parameterise the standard deviation statistic with an equation, it is not necessary to the simulation-optimisation approach described in the previous chapter. Additionally, since it is known that the standard deviation is a concave function, it is possible to use the ε-constraint method to derive the Pareto set.

In this manner, the problem can be solved with multiple runs of a single objective mathematical program if one objective is held fixed while the other is optimised as follows:

Minimise: $$\mathbf{Z}(\mathbf{x}) = \sigma_{Port} = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_i \sigma_j \rho_{ij} \right]^{1/2}$$

Where the objective function Z(X) denotes the standard deviation of the portfolio.

Subject to: $$\mathbf{g_1}(\mathbf{x}) = C_{Port} = \sum_{i=1}^{N} x_i Cost_i = Budget_P \qquad \textbf{Equation 5-1}$$

$$\mathbf{g_2}(\mathbf{x}) = E(NPV_p) = \sum_{i=1}^{N} x_i E(NPV_i) = T_{NPV} \qquad \textbf{Equation 5-2}$$

$$0 \leq \mathbf{X} \leq 1 \qquad \textbf{Equation 5-3}$$

Where $Cost_{port}$ is the investment cost of the portfolio, $Cost_i$ is the investment cost of the ith-project, $Budget_{port}$ is the overall capital constraint and T_{NPV} is a target value for the return. The first restriction (equation 5-1) is the total cost of the portfolio that has to be equal to the capital constraint. The second restriction (equation 5-2) is the expected return of the portfolio, which is stated to be equal to a target value T_{NPV}. This restriction is crucial because it allows defining the problem as single objective. The third restriction (equation 5-3) states that the working interest or participation levels must be a real number equal or greater than zero and less or equal than one.

[25] Under the assumption that the standard deviation or the variance of the returns is a risk measure. Alternatively, the problem can be re-stated to minimize the semi-standard deviation of the attribute under study.

5.3.2.2 Example

In order to solve the hypothetical portfolio optimisation problem presented at the beginning of this section with the Markowitz approach it is necessary to state the problem as follows:

For r =0 to r =N-1:

Minimise: $$\mathbf{Z}(\mathbf{x}) = \sigma_{Port} = \left[\sum_{i=1}^{8} \sum_{j=1}^{8} x_i x_j \sigma_i \sigma_j \rho_{ij} \right]^{1/2}$$

Subject to: $$\mathbf{g_1}(\mathbf{x}) = C_{Port} = \sum_{i=1}^{N} x_i Cost_i = US\$400 million$$

$$\mathbf{g_2}(\mathbf{x}) = E(NPV_p) = \sum_{i=1}^{N} x_i E(NPV_i) = Min(E(NPV)) + r$$

$$0 \le \mathbf{X} \le 1$$

Where the Min(E(NPV)) and Max(E(NPV)) are respectively the minimum and the maximum expected values of the NPV for the given level of budget and set of project proposals and “r” is an increment defined by [Max(E(NPV)) - Min(E(NPV))]/N where N is the number of efficient solutions that will be calculated for the efficient set. The presence of the “For” statement at the beginning of the optimisation shows the need for several runs of the optimisation problem when the ε-constraint method is used.

Figure 5-1 shows the calculated efficient frontier with N=20 for the set of projects and the given budget constraint of US$400 million. The leftmost portfolio in Figure 5.1 represents the least risky portfolio with an E(NPV) of US$60 million and SD(NPV) of US$18.78 million. The rightmost portfolio is the one with highest E(NPV) (US$100 million) and SD(NPV) (US$40.21).

Figure 5-2 shows the composition of the portfolios shown in the efficient frontier. It is possible to note that the rightmost portfolio is the same portfolio suggested by the capital rationing method since it maximises the E(NPV) metric for the same budgetary constraint.

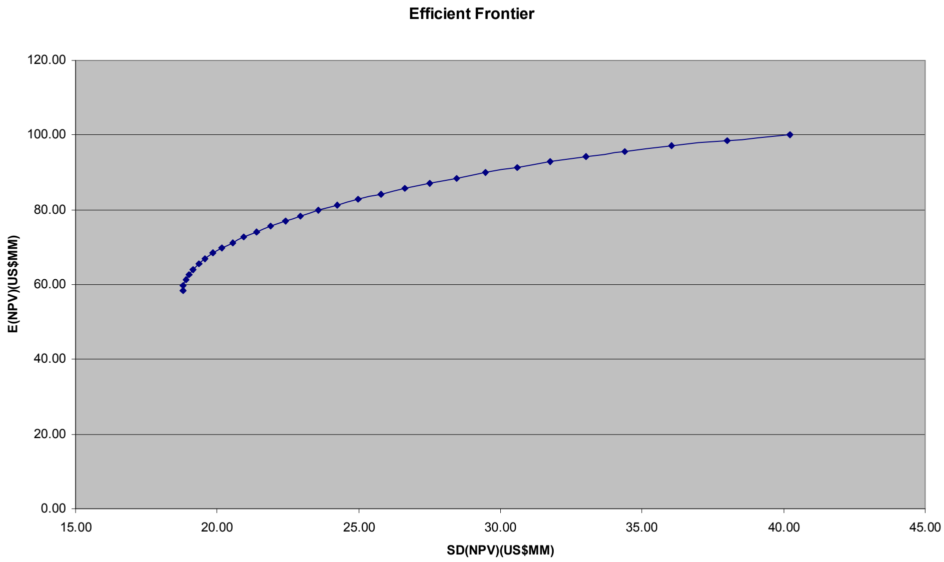

Figure 5-1 Efficient frontier calculated for the eight project proposals and a budget of US$400 million.

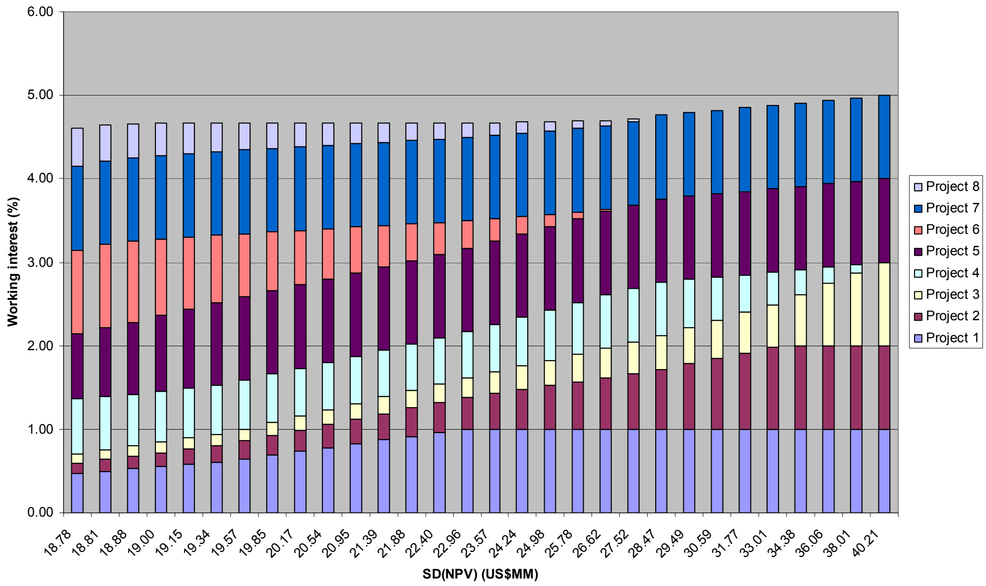

Figure 5-2: Working interest composition of the portfolios shown in the efficient frontier

The main advantages and disadvantages of the mean-variance method can be summarised as follows:

Advantages:

- Incorporates uncertainty and/or risk to the portfolio selection process and hence it is suitable to risk-averse companies.

- Incorporates correlations between projects and hence takes advantages of the risk reduction properties of diversification.
- Posterior articulation of preferences, hence DMs can explicitly trade-off the perceived utility of each potential investment strategy.
- Calculations can be solved using simple optimisation tools as Solver™ [26], which is an optimisation algorithm that is embedded in Excel™ [27].

Disadvantages:

- Although this method accounts for two objectives, these objectives are set over a single attribute. Hence, the performance in other attributes is a by-product of the risk-return of the NPV attribute.
- It is only possible to use this method with closed-form "risk/uncertainty" statistics such as the standard deviation, the semi-standard deviation or the variance.
- It does not account for the "time to invest" decision variable.

5.4 Review of the state of the art multi-attribute E&P portfolio optimisation methods

This section reviews three alternative approaches to the capital rationing and the mean-variance methods to optimise E&P portfolios whilst accounting for multiple performance attributes and uncertainty. These approaches use multi-attribute utility theory, goals over expected values and simulation optimisation respectively.

5.4.1 Multi-attribute utility theory (MAUT) approach

This method is described in Walls (1995) and draws on the multi-attribute utility theory literature to develop a decision model to select portfolios that explicitly maximises the expected utility of the corporation. The method proposes a full probabilistic characterisation of the project proposals in terms of the relevant attributes using Monte Carlo simulation.

After the individual projects have been evaluated, the method proposes the generation of 2500 possible portfolios based on different combinations of working interest that must be also characterised stochastically using Monte Carlo simulation. Then single attribute utility functions relating the utility perceived by the DM to each one of the performance attributes must be first elicited from the DM. A critical part of the elicitation is to provide the DM with the overall minimum and maximum performance values of each one of the attributes for the 2500 portfolios generated. Once these single attribute utility

[26] Solver™ is a trademark of Frontline Systems.
[27] Excel™ is a trademark of Microsoft.

functions have been elicited they are normalized so that the minimum utility for each attribute is zero and the maximum utility is one. In this manner, it is possible to calculate the expected utility of each one of the portfolios generated for each one of the attributes under study. This can be expressed mathematically as follows:

$$E(u_i(z_i)) = \sum_{j=1}^{n} p_j u(z_j)$$

Where z_i is the attribute of interest and p_j is the probability of outcome j for each attribute z_i.

In order to aggregate the expected utilities for each of the attributes the method proposes the use of an additive multi-attribute utility function similar to the one presented in chapter 3 for the two attribute case. Hence, for an "n" number of attributes, this function can be represented mathematically as:

$$u(z_i) = \sum_{i=1}^{n} k_i E(u_i(z_i))$$

Where k_i is the elicited relative importance given by the DM to attribute z_i and $\sum_{i=1}^{n} k_i = 1$. In this manner the utility of each one of the 2500 portfolios is calculated and the one with the highest utility is the one that should be selected.

Advantages of the MAUT approach:

- The process of eliciting the single attribute and multi-attribute utility functions from the DMs forces them to think about their preferences which can be a source of insight to make a better decision.
- The process is mathematically simple and its application does not need a complex optimisation algorithm.

Disadvantages of the MAUT approach:

- The approach assumes the additive independence condition holds. However, according to Clemen and Reilly (2001) the additive independence condition rarely holds.
- The approach provides the optimum portfolio from the 2500 portfolios generated but fails to address if these 2500 portfolios are Pareto optimal, and moreover fails to address if these 2500 solutions are sparsely spaced over the range of the Pareto optimal set. In other words the DM may be assigning maximum or minimum utility to values that do not represent the actual maximum or minimum performance of the set of projects in a given attribute.

- The approach neglects the presence of inter-project correlations.
- The approach uses prior articulation of preferences and hence prevents the DM from explicitly understanding the tradeoffs among the attributes.

5.4.2 Multiple goals approach

This approach is presented in DuBois and Howell (2000); DuBois (2001); Howell and Tyler (2001) and Allan (2003). A very similar method is used by the software Capital Planning™[28] (Schlumberger (2002)). The main objective of this approach is to meet performance targets set by the DM over multiple attributes for several years into the future. The optimisation algorithm seeks for portfolios capable of meeting these targets on an expected value basis while minimising the risk of the portfolio. In this method the portfolios are defined in terms of working interest and time to invest.

This approach starts by characterising the project proposals using a semi-stochastic approach. Hence, instead of producing full probability distribution profiles of the performance attributes using Monte Carlo simulation, projects are characterised using a reduced number of scenarios of production and cost profiles. More specifically, Allan (2003) states that these scenarios usually correspond to the production of the P10, P50 and P90 volumes of the EUR, where each scenario is associated to a cost profile representing the capital disbursements required to produce those volumes. Once these profiles are built it is possible to calculate an approximate mean profile using the Swanson's rule[29]. Then these "mean" profiles of the production and the costs can be combined with the oil price to produce other financial profiles as income or profit.

Although unfortunately the literature describing this method is not clear about how the optimisation is actually performed, it is clear that the main interest of the method is to find portfolios meeting all the targets set by the DM. In this manner, the optimisation algorithm used by the method uses combinations of working interest and time that respectively "strech" and "shift" the mean profiles calculated with the use of the Swanson's rule to meet the performance targets.

DuBois (2001) states that this methodology cannot directly influence the probabilities of achieving the stated targets. The author explains that after a portfolio that meets the targets (modelled as constraints in the optimisation algorithm) is found, it is possible to perform a Monte Carlo simulation of the optimal solution and hence study the probabilities of meeting the targets. As a result, these probabilities are by-products of the optimisation approach since it is not possible to use them as part of the objective function or the set of constraints.

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[28] Capital Planning is a trademark of Schlumberger information solutions.

[29] Swanson's rule consists in assigning a 30% probability to the P10 profile, a 40% probability to the P50 profile and a 30% probability to the P90 profile. The weighted average of these factors is commonly used as a rough estimate of the mean.

As a consequence, it is possible to infer that this approach is based in the ε-constraint method described in the previous chapter. What is not fully clear about it in the literature is if the algorithm performs the optimisation in one step for every solution in the efficient frontier. In other words, the algorithm may either:

- Firstly find a set of feasible solutions on an expected value basis without the use of an objective function, secondly perform a Monte Carlo simulation of the feasible portfolios found and thirdly sort these solutions according to a risk-return dominance criteria.
- Firstly find a set of feasible solutions with an objective defined over the expected value of the NPV attribute and then perform steps two and three as in the previous item.
- Firstly find a single feasible solution with an objective defined over a risk metric (e.g., standard deviation) including an additional constraint for the NPV and secondly perform a Monte Carlo simulation of each of the solutions found.

However the 3rd strategy seems to be the less probable given the fact that DuBois (2001) states that the approach uses a risk metric called "mean loss" that is calculated while a Monte Carlo simulation is being performed. However, since this method is not capable of directly influencing probabilities, the use of a simulation-optimisation approach is discarded and hence the third strategy described, to the knowledge of the author of this thesis, is improbable.

The ε-constraint method is guaranteed to find a Pareto optimal solution if an objective function is set over one of the attributes and constraints are applied to the rest of attributes into consideration. However, if the method described in this section uses one of the two strategies described above the solutions found are not necessarily Pareto optimal in terms of risk and return since the risk metric used may not included as part of the set of constraints or the objective function.

Allan (2003) states that the model allows the use of a stochastic oil price as a global variable when the Monte Carlo simulation is performed. In this manner, the method accounts for stochastic inter-project correlation.

Advantages of the multiple goals method:

- Optimises both working interest and time.
- Accounts for intra-project and inter-project correlation.
- Accounts for multiple attributes.
- Accounts for the uncertainty in the performance of the attributes.
- Includes the time-dimension to the optimisation procedure.

Disadvantages of the multiple goals method:

- The multiple performance attributes are included through the set of constraints. Consequently even if the solutions found are Pareto optimal, it is not possible to explicitly study the trade-offs among the attributes.
- The existence of tradeoffs among the attributes may prevent the DM from finding a feasible solution for a given set of goals. This may then force the DM to change the goals of the portfolio on an iterative basis or bring new potential projects to the optimisation until a feasible solution is found. However, this approach concentrates more on feasibility than on optimality. Hence it finds solutions that satisfy the DM goals but not necessarily delivers the maximum potential of the set of projects in terms of the preferences of the DM.
- Although it is not possible to assure this with the literature available, it is probable that the solutions found by this approach are not Pareto Optimal.

5.4.3 Single objective simulation-optimisation approach

As discussed in chapter 3, the simulation optimisation approach combines Monte Carlo simulation with meta-heuristic optimisation methods. The main advantage of this approach is possibility of including statistics, percentiles and/or probabilities of achieving a certain target as part of the objective function or the set of constraint. According to Campbell et al. (2001) there are at least two commercially available softwares that use this methodology named Optquest™[30] and RiskOptimizer™[31].April et al. (2003); Rodriguez and Galvao (2005) show how the simulation optimisation software Opquest™ can be used to solve E&P portfolios.

April et al. (2003) show two simple example cases where it is possible two set various statistics as part of the objective function and the set of constraints. In the first example the authors set a E&P portfolio optimisation problem where the DMs wish to construct a mean-standard deviation efficient frontier. The problem is set so that the objective function is the E(NPV) of the portfolio, and there is a constraint set over the SD(NPV). In this case the authors just consider the working interest vector as the set of decision variables. The efficient frontier can be constructed using the ε-constraint method varying the level of the constraint set over the standard deviation.

The second case presented in April et al. (2003) shows a more unusual case for the E&P portfolio optimisation literature. In this example the DMs are interested in maximising the chance of getting a NPV larger or equal than a given target while keeping the P10(NPV) to be larger than another target.

[30] Opquest™ is a trademark of Decisioneering.

[31] Riskoptimizer™ is a trademark of Palisade

Additionally, the decision variables are set to be the working interest of each project and the starting year of each project.

Rodriguez and Galvao (2005) show a more detailed E&P portfolio optimisation problem with 8 projects of different levels of maturity and multiple performance attributes. The performance attributes used by the authors for this case include NPV, cash flow and production rates. In this manner, every project is stochastically characterised from reserves to economics. Additionally, the oil price is set as a stochastic global variable creating correlation through the project proposals.

It is important to note that in this method the performance of the portfolio is characterised not only with summary attributes (e.g., NPV, reserves) but with yearly production and after tax net cash flow profiles. Once the projects are characterised the authors set the E(NPV) of the portfolio as the objective to be maximised while setting constraints over the expected values of the yearly production and cash flow profiles. Additionally, the decision variables are set to be the working interest of each project and the starting year of each project. Consequently, this approach is very similar to the multiple goals approach previously described but with the capability of optimising statistics, percentiles and probabilities of achieving targets.

However, the fact that this method allows us to work with uncertainty and risk measures adds another level of complexity to the problem since then it is necessary to somehow account for intra-project correlations. Rodriguez and Galvao (2005) sort out this issue by integrating the project characterisation with the portfolio optimisation. In this manner since the set of equations that describe the relationships among the various attributes of interest for each project are embedded in the model it not necessary to model these correlations explicitly[32].

The approaches of April et al. (2003); Rodriguez and Galvao (2005) show how the simulation-optimisation approach can help overcome some of the disadvantages showed by the multiple-goals approach described in the previous section. The main advantage over the multiple-goals approach is that since all the objectives set in the form of statistics are included in the optimisation problem via the objective function or through a set of constraints, if a feasible solution is found this solution is Pareto optimal. However, this methodology still lacks the capability of producing Pareto efficient sets for multiple (>2) objectives and hence prevent the DM to explicitly understand the tradeoffs about their multiple objectives. The relevance of this statement is recognised in Rodriguez and Galvao (2005) in the following way:

“Sometimes the requirements (constraints over attributes) may conflict with each other or lead to inefficient portfolio management, destroying value and elevating the risk.”

[32] This issue will be discussed in more detail in the next chapter.

It is possible to infer from this statement that even if an efficient feasible solution is found, it may not maximise the utility of the DM (and hence perceived utility of the shareholders). Consequently, a methodology capable of producing a Pareto set would help DMs to make better decisions.

Advantages of the single objective simulation-optimisation approach:

- Capable of accounting for working interest and time as decision variables.
- Accounts for intra-project and inter-project correlation with the use of Monte Carlo simulation.
- Accounts for multiple attributes.
- Accounts for the uncertainty in the performance of the attributes.
- Capable of setting targets and constraints over statistics, percentiles or probabilities over the performance attributes.
- Includes the time-dimension to the optimisation procedure with the use of yearly profiles of the portfolio performance attributes.

Disadvantages of the single objective simulation-optimisation approach:

- The multiple performance attributes are included through the set of constraints. Consequently even if the solutions found are Pareto optimal, it is not possible to explicitly study the trade-offs among the attributes.
- Similarly to the multiple goals approach this approach concentrates more on feasibility than on optimality. Hence it finds solutions that satisfy the DM goals but not necessarily delivers the maximum potential of the set of projects in terms of the preferences of the DM.
- A major disadvantage of the simulation optimisation approach not explicitly mentioned in the literature is that it is considerably slower than the other approaches since it has to perform a simulation of each of the solutions proposed by the meta-heuristic search method used.

5.5 Summing up

Firstly, this chapter described how to aggregate the individual performance attributes deterministically and stochastically to assemble a portfolio of project proposals. Secondly, it described the capital rationing method and mean-variance method to optimise E&P portfolios. It was shown that the main advantages of the mean-variance approach over the capital rationing approach is the inclusion of risk or uncertainty metrics into the optimisation and the capability to explicitly show the tradeoffs between risk and return. It was also shown that the major flaw of the mean-variance approach is its inability to account for multiple attributes and its inability to account for risk metrics that cannot be expressed in a closed form.

Lastly, three alternative methods found in the literature to account for multiple attributes and risk were described. These methods are the MAUT approach, the multiple goals approach and the simulation-optimisation approach. It was shown that the simulation optimisation approach can be seen as an extension of the multiple goals approach but with the possibility of optimising multiple statistics percentiles and probabilities from the PDFs that characterise a project investment portfolio. However, these methods account for several performance attributes they are unable to produce a Pareto surface explicitly showing the tradeoffs among the multiple attributes.

6 A MULTI-OBJECTIVE SIMULATION-OPTIMISATION APPROACH FOR E&P PORTFOLIO OPTIMISATION

6.1 Introduction

The previous chapter showed that the state of the art methods found in the E&P portfolio selection literature are not capable of producing a set of Pareto optimal portfolios in terms of multiple objectives set over statistics, percentiles and probabilities calculated from multiple attributes. This chapter proposes an E&P portfolio optimisation model to solve problems with the following characteristics:

- Multiple attributes with uncertain performance.
- Multiple stochastic objectives per attribute (in the form of statistics, percentiles and probabilities of achieving certain targets).
- Posterior articulation of preferences.
- Partial funding.
- Intra-asset dependencies among performance attributes.
- Inter-asset dependencies among performance attributes.
- Linear budget constraint.

The proposed model uses a simulation-optimisation approach where the valuation of individual potential assets is integrated into the portfolio optimisation problem to assure the preservation of intra-asset and inter-asset dependencies. The model uses a new version of the multi-objective genetic optimisation with linear constraints (MOGOL) algorithm presented in chapter 3 as search engine. The output of the model is an approximate set of Pareto optimal projects in terms of all the objectives under consideration, where each performance attribute has one or more stochastic objectives.

Since the portfolio selection approach proposed here uses the model to characterise each asset as part of the portfolio optimisation problem, the first part of the chapter describes how the individual projects are modelled. The second part of this chapter explains how the individual projects are assembled into a portfolio model and how the search engine MOGOL was linked to the portfolio model.

6.2 Single project model

The ultimate objective of the single-project model here presented is to produce distributions of the attributes reserves and NPV for each of the project proposals that will be used in the optimisation.

6.2.1 Production profile model

The portfolio optimisation model presented in this chapter requires a stochastic production profile for each of the projects under study. This stochastic production profile is generated running a Monte Carlo simulation over a model that replicates the behaviour of an oil reservoir. It is assumed that the each project is designed to produce the oil of geologically independent reservoirs.

Lund (1997) states that there are relatively few models that address the complete development project under uncertainty as a consequence of the massive amount of computer power needed to do this. Therefore, including uncertainty in such a model will clearly enhance this problem. However, the same author states that given the high degree of uncertainty in the early stages of development it is doubtful that a detailed asset description will provide better insight about its future performance than a coarser model. This issue is particularly relevant for the model proposed in order to get reasonable solution times. Since the proposed model is based in a simulation-optimisation approach and the objectives are statistics read from the distributions calculated after each simulation, it is necessary to keep the model simple to get a good approximation of the true value of these statistics with the minimum number of iterations per simulation.

As a consequence of the above, each one of these reservoirs is assumed to be perfectly homogeneous. According to Lund (1997) this type of reservoir model is commonly termed a tank model. Since this type of model assumes a reservoir without spatial variations, the locations of the wells are of no consequence to the production and hence, it would be possible to deplete the reservoir from a single well. This "zero dimensional" reservoir model, commonly called "tank model" and widely applied in analysis of field developments (Lund (2000)) is used here. This model rests on the following assumptions (Lund (2000)):

$$P_{w,t} = P_{w,0} - \frac{R_0 - R_t}{R_0}\left(P_{w,0} - P_{\min}\right) \qquad \textbf{Equation 6-1}$$

and,

$$q_{r,t} = N_t q_{w,t}\left(\frac{P_{w,t} - P_{\min}}{P_{w,0} - P_{\min}}\right) \qquad \textbf{Equation 6-2}$$

Where:

$P_{w,0}$: initial well (reservoir) pressure.

$P_{w,t}$: well (reservoir) pressure at time t.

P_{min}: abandonment pressure.

R_0: initial reservoir volume.

R_t: remaining reservoir volumes at time t.

$q_{r,t}$: maximum reservoir depletion rate at time t (productivity of the reservoir at time t)

$q_{w,0}$: initial well rate.

N_t: number of producing wells at time t.

Equation 6-1 states that the pressure of the reservoir drops linearly with accumulated production. Equation 6-2 gives the proportional relationship between the number of producing wells, the well rates and the relative well pressure above minimum. Combining equations 6-1 and 6-2 it is possible to restate the maximum reservoir depletion rate as follows:

$$q_{r,t} = N_t q_{w,0} \frac{R_t}{R_0}$$

Where,

$$R_t = R_0 - \sum_{1}^{t-1} q_t$$

Assuming that the wells can produce at their maximum initial production rate without damaging the reservoir, the production of the field at time t (q_t) can be described as follows:

$$q_t = \min\{q_{r,t}, \text{MaxCap}\}$$

Since the reservoirs have not been produced before the existence of these developments it is possible to state that the initial reservoir volume (R_0) will be equal to the expected ultimate recovery (EUR). Hence, q_t can be described as:

$$q_t = \min\{N_t q_{w,0} \frac{R_t}{EUR}, \text{MaxCap}\}$$

Note from the previous equation that, the maximum depletion rate of the reservoir will be determined by the production capacity of the production hub (MaxCap) and the productivity of the reservoir ($q_{r,t}$). The maximum capacity (MaxCap) is a deterministic variable since the development plan of the project has been approved. On the other hand, the reservoir production ultimately depends on the initial rate per well ($q_{w,0}$), and the expected ultimate recovery (EUR) and these two variables are stochastic. It is assumed that all the wells start production at the same time and that all the wells have the same initial rate.

It is important to note that usually a larger reservoir will have a higher initial rate and hence there is a positive stochastic correlation between these two variables. Since these two variables are inputs of the model, it is necessary to model this dependency explicitly with the use of the Spearman rank order correlation coefficient (Murtha (2000)). In this way, the Monte Carlo simulation engine will relate the sampling of high values of the variable EUR to high values of ($q_{w,0}$) and the sampling of low values of EUR to low values of ($q_{w,0}$). A coefficient of 50% is used for all the project proposals modelled.

The total cumulative production at time t (Q_t) is defined as follows:

$$Q_t = \sum_{t'=0}^{t} q_{t'}$$

6.2.2 Oil price model

The model assumes a mean-reverting behaviour in the oil prices. Hence the mean-reverting model explained in chapter 3 is used. For simplicity, all calculations are done in real terms and hence oil prices are not escalated to account for inflation.

6.2.3 Costs

The model includes capital expenditures (Capex), operating expenditures (Opex) and abandonment expenditures (Abex). As mentioned above, for simplicity, all calculations are in real terms and hence costs are not inflated[33].

6.2.3.1 Capex

Since all the projects are assumed to be in the "under development" or "approved for development"[34] stages, the Capex is modelled as a deterministic value[35]. For simplicity, it is also assumed that all the Capex is spent in the year before the project starts production.

For tax purposes the total Capex is subdivided in intangible Capex (IntanCapex) and tangible Capex (TanCapex). The fiscal regime assumed for the model states that the intangible Capex are expensed and that the tangible costs are capitalised using four-year straight line depreciation.

6.2.3.2 Opex

The Opex of each project proposal is defined as follows:

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33 Ignoring inflation is obviously a simplification. In an non academic context, inflation must be acknowledged in order to produce accurate valuations.

34 Further research in this area including a mix of projects at various development stages should provide a more realistic illustration of the upstream oil and gas portfolio optimisation decision setting.

35 The fact that there is a commitment to develop a project does not imply that there is no uncertainty associated with the Capex variable. However, at this development stage, the uncertainty regarding the Capex is by definition significantly lower than in an exploration/appraisal project.

$$\text{OpEx}_t = \text{FixOpEx}_t + q_t \cdot \text{VarOpEx}_t$$

Where FixOpEx_t is the fixed Opex and is modelled as a deterministic variable and VarOpEx_t is the variable Opex and is modelled as a uniform distribution. The fiscal regime used assumes that the Opex are expensed.

6.2.3.3 Abex

The abandonment cost of the project proposals is modelled as a triangular distribution. The main reason for this is that since it is an expenditure that will occur many years ahead the starting of the project, its total amount is uncertain. Additionally, its date of occurrence is also uncertain since it depends on the economic limit[36] of each of the projects. Its date of occurrence is modelled as follows:

If $q_{t,econ}=0$ and $\sum_{0}^{t-1} q_{t,econ} > 0$ then $\text{Abex}_t=\text{Abex}$

Otherwise: $\text{Abex}_t=0$

Where $q_{t,econ}$ is the production profile that is economically feasible to produce[37], Abex is the magnitude of the abandonment costs defined as a stochastic variable and Abex_t is the abandonment costs incurred in year t. The abandonment costs are assumed to be incurred in a single year.

6.2.4 Tax calculation

For simplicity, the model assumes that all projects are implemented in countries with simple concessionary regimes. The main reason for this assumption is to save computing power since the modelling of production sharing contracts (PSC) usually requires the use of lookup tables and more complex calculations than a concessionary regime and consequently the model would become slower. Although the value of the parameters of the concessionary fiscal regime used may change from project to project, the structure of the fiscal regime is kept equal for all the projects. The structure of the fiscal regime is as follows:

$$GrossRevenue_t = q_t \cdot P_t$$

$$Royalty_t = GrossRevenue_t \cdot RoyaltyRate_t$$

$$NetRevenue_t = GrossRevenue_t - RoyaltyRate_t$$

36 This term will be discussed in more detail later in this chapter.
37 This term will be discussed in more detail later as part of the economic limit concept.

If $IntanCapex_t + Opex_t + DD\&A_t + TaxLossCF_t \geq NetRevenue_t$,

then $TotalAppliedDeductions_t = NetRevenue_t$

Otherwise:

$$TotalAppliedDeductions_t = IntanCapex_t + Opex_t + DD\&A_t + TaxLoss_t$$

It is possible to note that in the previous equation, the abandonment costs are not entering the calculation for the reasons explained in the previous section.

Additionally, if: $TaxableIncome_{t-1} < 0$ then, $TaxLossCF_t = -TaxableIncome_t$

Where:

$$TaxableIncome_t = NetRevenue_t - IntanCapex_t - Opex_t - DD\&A_t - TaxLossCF_t$$

Additionally, if:

$TaxableIncome_t > 0$ then $IncomeTax_t = TaxableIncome_t \cdot TaxRate$

Otherwise:

$$IncomeTax_t = 0$$

Finally:

$$BTNCF_{t,oper} = NetRevenue_t - IntanCapex_t - TanCapex_t - Opex_t$$

$$ATNCF_{t,oper} = BTNCF_{t,oper} - IncomeTax_t$$

Where $DD\&A_t$ is the depletion, depreciation and amortization in time t and $TaxLossCF_t$ is the tax loss carried forward in year t, and $BTNCF_{t,oper}$ and $ATNCF_{t,oper}$ are, respectively, the before tax net cash flow and after tax net cash flow in time t that is operationally feasible. The term "operationally feasible" means that it is physically possible to produce these cash flows. However, these cash flows do not account for the economic criteria to terminate the operations of the field when the operating cash flow of the field becomes negative on a steady basis.

6.2.5 Reserves and NPV calculation

The reserves of the field will be equal to the total cumulative production of the field from its beginning until its abandonment. The abandonment date is defined by the time in which the field production becomes unfeasible economically. This date is commonly called "economic limit".

In this model the economic limit of the project is defined as follows:

If $BTNCF_{t,oper} < 0$, and $\sum_{0}^{t-1} BTNCF_{t,oper} > 0$ then $t = t_{EconLimit}$

Consequently, $q_{t,econ} = 0 \forall t > t_{EconLimit}$, otherwise $q_{t,econ} = q_t$.

Where $q_{t,econ}$ is the economically feasible stream of production in time t. In other words, if the field has been producing a positive operating cash flow and this cash flow declines until becoming negative, then the field must be closed and hence there is not any more production from that point forward.

Given that the reserves of a project are those resources that are **operationally** and **economically viable to produce**, the reserves of the field R are defined as follows:

$$R = Q_{EconLimit,oper} = \sum_{0}^{t_{EconLimit}} q_{t,oper}$$

Hence, $BTNCF_{t,econ} = 0 \ \ \forall t > t_{EconLimit}$ and $BTNCF_{t,econ} = BTNCF_{t,oper} + Abex_t \ \ \forall t \leq t_{EconLimit}$.

Additionally, $ATNCF_{t,econ} = BTNCF_{t,econ} - IncomeTax_t$

Consequently, the net present value of a project is calculated as follows:

$$NPV = \sum_{0}^{t=EconLimit} \frac{ATNCF_{t,econ}}{(1+r)^t}$$

Where r is the discount rate used by the company.

Summarising, this section described how to calculate the attributes "'reserves" and "NPV" for each one of the projects to be considered in the portfolio optimisation. Since the inputs to calculate these attributes are modelled as distributions, it is possible to run a Monte Carlo simulation of the model and, hence, obtain probability density functions of these two attributes.

6.3 Stating the objectives of the portfolio

As it was explained in chapter 5, with the exception of capital efficiency attributes, the performance of a portfolio of N assets on the performance attribute (Z_{Port}) can be expressed as linear combinations of the same attribute for each of the project proposals (z_i) proportioned by the working interest of the company on each project (x_i). Although the methodology here proposed is able to optimise any statistic, percentile or probability of any attribute expressed as a PDF, for illustrative purposes it will be assumed that the DMs are interested in optimising statistics of the reserves (R_{port}) and net present value (NPV_{port}) portfolio attributes. These attributes are represented mathematically as follows:

$$R_{Port}(\mathbf{x},\omega) = \sum_{i=1}^{5} (x,\omega)_i R_i$$

$$NPV_{Port}(\mathbf{x},\omega) = \sum_{i=1}^{5} (x,\omega)_i NPV_i$$

Where ω represents the stochastic nature of the variables. In the same manner, the capital investment necessary to fund the portfolio is defined as:

$$CapEx_{Port}(\mathbf{x}) = \sum_{i=1}^{N} x_i CapEx_i$$

Where $CapEx_{port}$ is a deterministic variable.

6.4 Integrated portfolio multi-objective optimisation model

The Markowitz approach reviewed in the last chapter clearly recognises the need to account for the stochastic dependencies among the net present values of the various projects included in the optimisation. However, in the same order of ideas it is necessary to recognise that the inclusion of the attribute “reserves” would necessarily require including not only the correlations among the attributes of the various project proposals but also the correlation among various attributes belonging to a single project.

Since it is assumed that the DMs are interested in optimising certain statistics of the attributes NPV and Reserves then it would be necessary to recognise that, for each one of the projects under study, the NPV attribute and the Reserves attribute would be structurally correlated as a consequence of the set of equations presented in the item 6.2. In other words, scenarios of high values in the NPV attribute will tend to show high values in the Reserves attribute and vice versa. This intra-asset correlation is represented in Figure 6.1 by the blue arrows.

Additionally, the model presented here assumes that the only source of correlation among projects is the oil price. Consequently, the oil price is modelled as a global variable in which the PDF that represents the forecast of prices for a given year is multiplied by the production in the same year of each one of the project proposals.

However, this feature will not only correlate the NPVs of the various projects involved, the attribute Reserves in each one of the projects is also influenced by the oil price via the economic limit (a higher oil price will push the economic life of the project into the future and hence more hydrocarbons may be produced). Hence, it is possible to infer that not only there are correlations among the different attributes of a single project (blue arrows in Figure 6.1) and among the net present values of the various projects under consideration as the Markowitz approach suggests (black arrow in Figure 6.1) but that there might also exist correlations among all the attributes of all projects (red arrows in Figure 6.1).

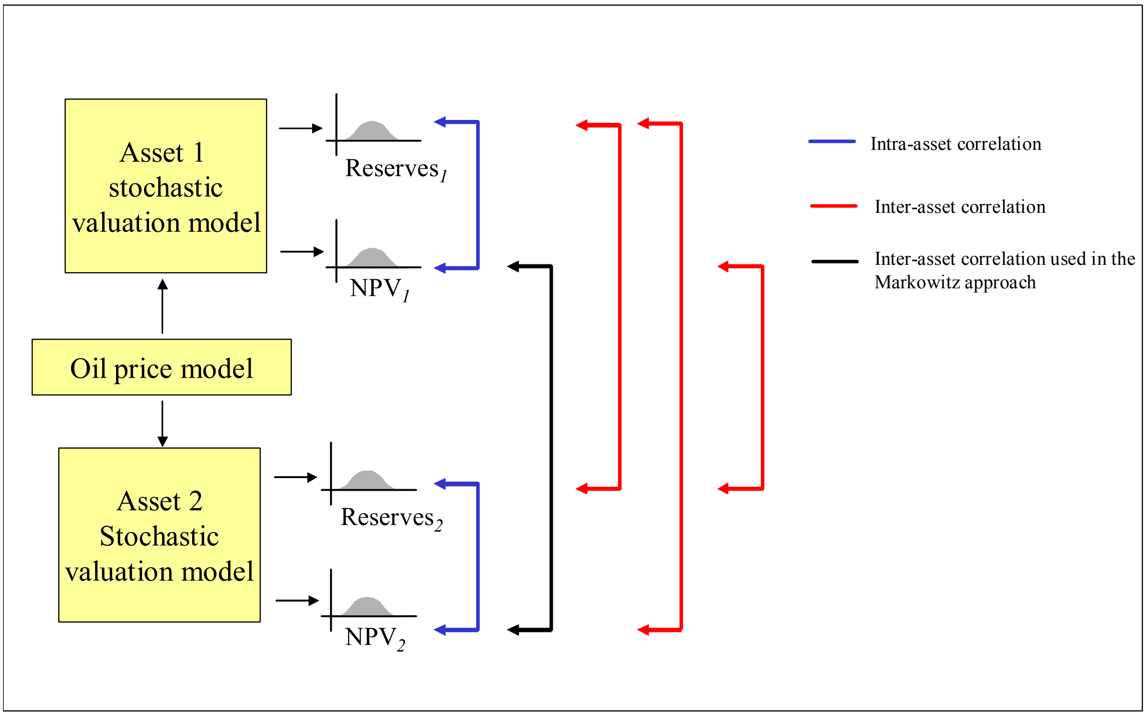

Figure 6-1 Intra and inter-asset correlations

6.4.1 Integrating the search engine with the portfolio model

The multi-objective genetic optimizer with linear constraints (MOGOL) algorithm described in chapter 4 was selected as the optimisation engine that best suited the characteristics of the E&P portfolio problem proposed in this thesis. However, the initial version of MOGOL does not allow the inclusion of inter-project or intra-project correlations. Figure 6.2 shows that the initial version of MOGOL needed two steps. In the first step, the project proposals are characterised stochastically and PDFs are generated for each one of the attributes of interest. Then, in a second step, these PDFs are used as an input to the multi-objective simulation-optimisation approach where, as explained in chapter 4, an approximate Pareto surface is produced.

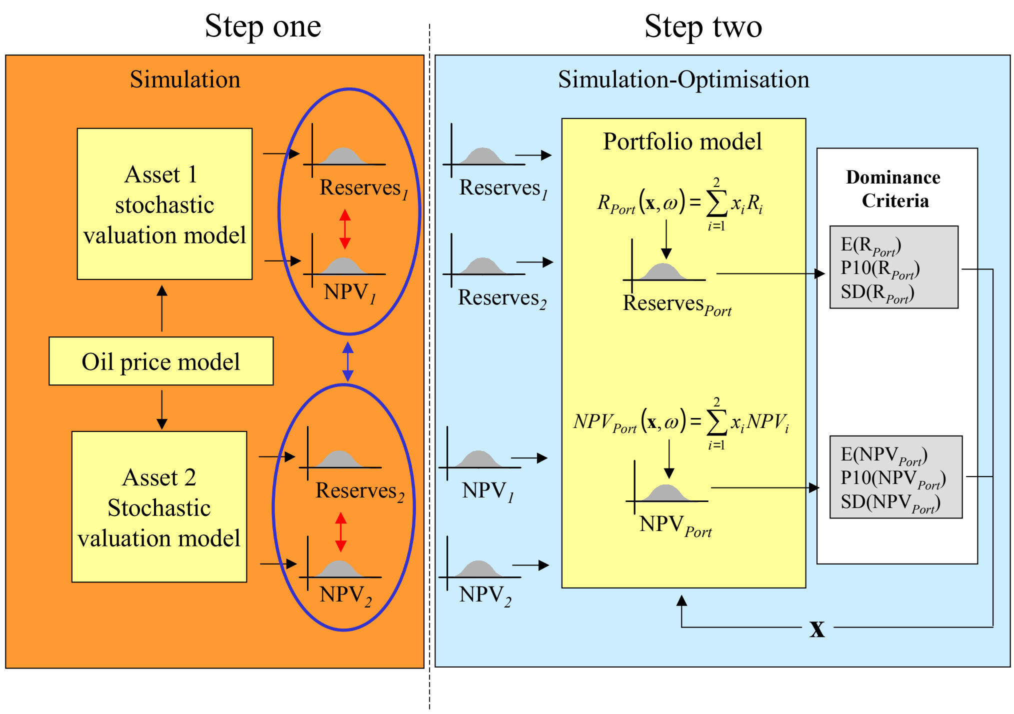

Figure 6-2 Separation of the project characterisation and portfolio optimisation stages

Note that this process does not honour the intra and inter-project correlations previously explained represented in Figure 6.2 by the blue ovals. Consequently, in order to account for these correlations the first version of the MOGOL was modified to work in an integrated manner with the portfolio model described in this chapter. Figure 6.3 shows how the PDFs used as an input to the optimisation model remain linked to their respective single project valuation models. In this manner each simulation performed for a solution proposed by MOGOL accounts for all the intra-project and inter-project correlations embedded in the portfolio model.

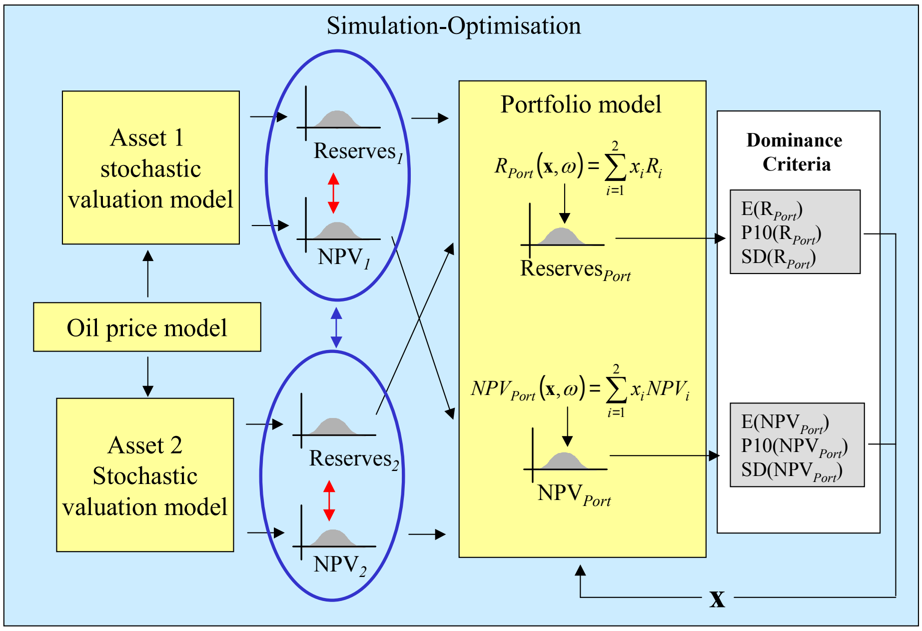

Figure 6-3 Integration of the project characterisation and portfolio optimisation stages

The project valuation models and the resulting portfolio model previously described in this chapter were modelled using Excel™ and Crystal Ball™. On the other hand, the initial version of MOGOL is an application coded in JAVA™[38]. Additionally, as it was described in chapter 4, the initial version of MOGOL has its own built-in Monte Carlo simulator. Hence, in order to integrate the Excel™ based portfolio model with the MOGOL it was necessary to adapt the MOGOL to interact with Excel™ and Crystal Ball™. In this manner the JAVA™ code was modified so that MOGOL proposes the solutions as explained in chapter 4 but instead of running the Monte Carlo simulation internally, loads a proposed solution in the Excel™ workbook with the portfolio model and calls Crystal Ball™ to run a simulation. Once the simulation is complete, MOGOL reads the relevant objectives from the worksheet and saves the results to be compared according to the dominance criteria explained in chapter 4. Each one of the solutions is treated in the same way until the number of generations set by the user is completed.

The new version of MOGOL is called MOGOL-XL and was co-developed with Professor Andres Medaglia[39] who is also the author of the initial algorithm. The main conceptual differences between the original version of MOGOL and MOGOL-XL were proposed by the author of this thesis. Coding of the MOGOL-XL was made by Professor Medaglia in JAVA™ with the exception of a Visual Basic for Applications™[40] routine coded by the author of this thesis to link MOGOL XL with Excel™ and Crystal Ball™.

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[38] JAVA is a trademark of Sun microsystems
[39] Andres Medaglia is an associate professor of the department of industrial engineering at the Universidad de los Andes, Bogota, Colombia. Email: amedagli@uniandes.edu.co
[40] Visual basic for applications is a trademark of Microsoft.

Note that the current version of MOGOL-XL is designed to deal with four stochastic objectives and only has maximising capabilities. Hence, with the current version it is not possible to minimise objectives (e.g., standard deviation).

6.5 Summing up

This chapter described the multi-objective portfolio selection model proposed in this thesis. The first part of the chapter described the modelling of the individual projects used as an input for the portfolio model. The main characteristics of these projects are:

- The production of these projects is forecasted using a “tank model” that assumes a zero dimensional reservoir where the pressure of the reservoir drops linearly with accumulated production.
- The oil price forecast is modelled with a mean reverting stochastic process.
- The projects are under development or approved for development and hence it is reasonable to model the initial capital outlay deterministically.
- The projects are located in countries with a simple concessionary fiscal regime.

The second part of this chapter described how the individual projects are aggregated to build the portfolio objectives. The importance of keeping the intra and inter-project dependencies was addressed as well as the consequent need to integrate the valuation model with the optimisation algorithm. Finally, the necessary changes made to the MOGOL algorithm to account for the portfolio optimisation problem proposed in this thesis were described.

7 APPLICATION OF THE MODEL TO A SET OF DEVELOPMENT PROJECTS

7.1 Introduction

This chapter applies the model presented in chapter 6 to a case of five offshore projects. The values used to build projects are hypothetical yet realistic. Two experiments are performed with these projects. The first experiment optimises 4 objectives set over two attributes. The second experiment optimises 4 objectives over one attribute. The results of both experiments are presented and discussed.

7.2 The projects

The example shown in this chapter uses five "approved for development" projects. There were two main reasons to keep the number of projects small:

- With a small number of projects it is easier to track the impact of a particular project in the various objectives of the portfolio.
- A small number of projects reduce the necessary computing time to perform the optimisation.

7.3 Experimental settings hardware and software

7.3.1 Hardware

All the experiments were run using a 2.0 GHz Pentium™ 4 laptop with 256MB of RAM under Microsoft Windows XP.

7.3.2 Software

MOGOL-XL was used as a multi-objective search engine. The stochastic single project models and the resulting portfolio were modelled using Excel™ 2003 and Crystal Ball™ 7.2.

7.3.3 Projects Inputs

The five projects were built using the "single project model" presented in the previous chapter. This model requires the following variables as inputs for each project.

7.3.3.1 Stochastic inputs

- *Initial production:* modelled as a triangular[41] distribution and hence requiring minimum, most likely and maximum values as inputs.
- *Estimated ultimate recovery:* modelled as a lognormal distribution and hence requiring the mean, the standard deviation and minimum and maximum values (optional) as inputs.

[41] Triangular distributions are rarely representations of the real world but are analytically convenient when the decision maker only have access to approximate low, high and most likely values describing the uncertainty of a variable.

- *Variable Opex:* modelled as uniform distribution and hence requiring minimum and maximum values as inputs.
- *Abex:* modelled as a triangular distribution.

7.3.3.2 Deterministic inputs

- *Total Capex*
- *Fixed Opex*
- *Maximum capacity of the facility of the field*
- *Discount Rate*
- *Number of wells*
- *% Intangible CAPEX (% of total Capex that is intangible)*
- *Tax Rate*
- *Royalty rate*

Table 7.1 and 7.2 summarise the input values used for each one of the stochastic variables for each of the five projects considered.

	Initial Production (Mbbls/d)			EUR (MMbbls)			
	Min	ML	Max	Mean	Std Dev	Min	Max
Project 1	4.5	5.0	6.0	330.0	150.0	100.0	700.0
Project 2	4.4	5.0	5.1	400.0	160.0	120.0	800.0
Project 3	4.0	4.5	5.0	100.0	40.0	25.0	200.0
Project 4	7.0	8.0	9.0	120.0	30.0	10.0	210.0
Project 5	4.0	4.5	5.0	200.0	50.0	80.0	360.0

Table 7-1 Summary of stochastic project inputs: initial production and EUR

	Variable Opex (MM$)		Abex (MM$)		
	Min	Max	Min	ML	Max
Project 1	1.0	1.5	135.0	150.0	165.0
Project 2	1.0	2.0	135.0	150.0	165.0
Project 3	1.0	2.0	67.5	75.0	82.5
Project 4	1.0	2.0	29.3	32.5	35.8
Project 5	1.0	2.0	121.5	135.0	148.5

Table 7-2 Summary of stochastic project inputs: variable Opex and Abex

Similarly, Table 7.2 and 7.3 summarises the input values for each of the deterministic variables for each of the five projects considered.

	Number of wells	CAPEX (MM$)	Fixed OPEX (MM$/y)	MaxCap (Mbbls/d)
Project 1	20.00	1500.00	100.00	80.00
Project 2	15.00	1500.00	80.00	40.00
Project 3	10.00	500.00	20.00	40.00
Project 4	10.00	650.00	20.00	50.00
Project 5	15.00	900.00	20.00	20.00

Table 7-3 Summary of deterministic project inputs number of wells, Capex, fixed Opex and maximum capacity

	Tax Rate (%)	Royalty Rate (%)	Discount rate (%)	% INTANGIBLE CAPEX
Project 1	40.00%	15.00%	10.0%	10.0%
Project 2	40.00%	12.50%	10.0%	10.0%
Project 3	30.00%	10.00%	10.0%	10.0%
Project 4	35.00%	12.50%	10.0%	10.0%
Project 5	35.00%	12.50%	10.0%	10.0%

Table 7-4 Summary of deterministic project inputs tax rate, royalty rate, discount rate and percentage of total Capex that is intangible

7.3.4 Mean reverting oil price model base parameters

The parameter values used for the mean reverting model described in chapter 3 are:

$P_{t=0}$ = 50$/bbl

M= 20$/bbl

Δt= 1 year

η=5years

Price floor=8$/bbl

σ=3$/bbl

Figure 7.1 shows the behaviour of the mean reverting oil price model for the inputs specified above. Since the stochastic quality of the model comes from the parameter ε, that is a normal distribution, the price for each year will also be normally distributed. Consequently, the P50 (median) of the distribution will be equal to the mean of the distribution.

For this reason it is possible to note how the P50 series in Figure 7.1 reverts to a long term mean of 20$/bbl as expected. This graph shows that for each year there is an 80% chance that the oil price will be between the P10 and the P90 series. Additionally, the green sample path shows one possible realisation of the model. This green sample path corresponds to a single iteration of a Monte Carlo simulation.

Figure 7-1 Behavior of the mean-reverting oil price model with the inputs specified above

7.3.5 Single project output

This section shows the performance of the five projects under study using the inputs shown in tables 7-1, 7-2, 7-3 and 7-4 and the oil price model shown in Figure 7-2. It is assumed that corporate DMs are interested in the attributes NPV and reserves.

Table 7-4 and table 7-5 summarise the performance of the 5 project proposals in terms of relevant statistics, probabilities and percentiles extracted from their NPV and reserves PDFs. These statistics, probabilities and percentiles are used as optimisation objectives at the portfolio level in the experiments performed later in this chapter. The full PDFs of the NPV and the reserves attributes and the P10, P50 and P90 net cash flow and production profiles of each project are shown in appendix C.

	E(NPV)	P(NPV>0)	P10(NPV)	P90(NPV)
	US$MM	%	US$MM	US$MM
Project1	223.5	68.1	-359.3	800.6
Project2	49.8	56.4	-257.8	371.7
Project3	223.5	86.2	-30.4	495.7
Project4	283.7	95.4	50.4	526.3
Project5	65.2	68.6	-94.1	229.5

Table 7-5 Summary of the performance of the projects on relevant statistics, probabilities and percentiles of the attribute NPV

	E(Reserves) MMbbls	P10(Reserves) MMbbls
Project1	105.2	59.6
Project2	121.9	74.2
Project3	34.1	20.3
Project4	42.9	30.8
Project5	70.0	50.2

Table 7-6 Summary of the performance of the projects on relevant statistics, probabilities and percentiles of the attribute reserves

7.4 Convergence of the objectives

Since the simulation optimisation approach strongly depends on the accuracy of each one of the Monte Carlo simulations performed during the optimisation, it is worthwhile to analyse the convergence of the simulations over each one of the objectives of interest. Crystal Ball™ 7.2 has a function called "bootstrap", this function estimates distributions of the statistics and percentiles of interests if a simulation is performed multiple times. Hence, these distributions give an indication of how accurate a simulation is. Since the experiments are set to use 2500 iterations per simulation, the bootstrap function will calculate the range of possible outcomes based on a simulation of 2500 iterations.

7.4.1 Convergence of the E(NPV), P10(NPV) and P90(NPV)

Table 7-7 shows that the expected value of the objective E(NPV) has a standard deviation of 13.9 millions and an expected value of US$ 865.7 millions. This means that around 67% of the time, the E(NPV) of the simulations will be inside a range of plus or minus 13.9 millions from the "true" E(NPV). Similarly, the P10(NPV) and P90(NPV) objectives have standard deviations of 23.8 millions and 27.5 millions respectively.

NPV (MM$)			
Statistic/Percentile	E(NPV)	P10(NPV)	P90(NPV)
Trials	200	200	200
Mean	865.7	-52	1,794.30
Median	864.6	-48	1,796.20
Mode	835.1	-34	1,801.90
Standard Deviation	13.9	23.8	27.5
Variance	193.9	568.5	757.2
Skewness	0.29546	-0.44494	-0.03001
Kurtosis	2.71	3.45	2.86
Coeff. of Variability	0.01609	-0.45838	0.01534
Minimum	835.1	-122.2	1,727.30
Maximum	900.9	5.6	1,864.00
Mean Std. Error	1	1.7	1.9

Table 7-7 Summary of the convergence of relevant statistics extracted from the NPV attribute

7.4.2 Convergence of the E(R) and P10(R)

Table 7-8 shows that the expected value of the reserves has a standard deviation of 1.2MMbbls and an expected value of 373.8 MMbbls. On the other hand the P10(R) objective has a standard deviation of 1.7 MMbbls.

Reserves (MMbbl)		
Statistic/Percentile	**E(R)**	**P10(R)**
Trials	200	200
Mean	373.8	298.3
Median	373.7	298.6
Mode	371.2	298.6
Standard Deviation	1.2	1.7
Variance	1.5	2.9
Skewness	0.22529	-0.23631
Kurtosis	2.88	4.51
Coeff. of Variability	0.00324	0.00573
Minimum	370.9	292.6
Maximum	377.2	303.7
Mean Std. Error	0.1	0.1

Table 7-8 Summary of the convergence of relevant statistics extracted from the Reserves attribute

It is important to note that although the objectives set over the reserves attribute have a reasonable convergence, the variability of the statistics extracted from the NPV attributes is considerably larger. It is possible to infer that all the uncertainties affecting the NPV of the portfolio model presented here prevent the objectives set over this attribute from converging with a reasonable number of iterations per simulation. Unfortunately, it is not practical to run the optimisation model presented here with a higher number of iterations since the running time can grow significantly. Hence it is important to keep in mind that the results of the objectives set over the NPV attribute shown in the next two sections will have a considerable range of error.

7.5 Experiment 1: Multiple objectives over multiple attributes

In this section the proposed portfolio optimisation model is applied to the five projects previously described. Since it is assumed that any transaction cost is already included in the Capex of each project, a company interested in pursuing all the available projects would need to expend a total amount of US$5.05 billion. However, it is assumed that the company has a capital constraint of US$2.5 billion and is interested in maximising the expected value of the net present value (E(NPV)), the probability of achieving a positive NPV (P(NPV>0)), the expected value of the reserves (E(R)) and the 10th percentile of the reserves[42], P10(R).This statement can be summarised mathematically as follows:

Maximise: $$[E(NPV_{Port}(\mathbf{X},\omega)), P(NPV_{Port}(\mathbf{X},\omega) > 0), E(R_{Port}(\mathbf{X},\omega)), P10(R_{Port}(\mathbf{X},\omega))]$$

[42] The 10th percentile of the attribute reserves represents a value that has 90% chance of being larger than the value obtained. Although the SPE definition of proved reserves accounts for several economical and technical factors, the 10th percentile of the attribute reserves could be interpreted as approximation of the proved reserves of the portfolio and/or asset.

Subject to:

$$CapEx_{Port}(\mathbf{X}) = \sum_{i=1}^{5} x_i CapEx_i \leq US\$MM\,2500$$

$$R_{Port}(\mathbf{X}, \omega) = \sum_{i=1}^{5} x_i R_i(\omega)$$

$$NPV_{Port}(\mathbf{X}, \omega) = \sum_{i=1}^{5} x_i NPV_i(\omega)$$

And, $0 \leq \mathbf{X} \leq 1$

Where $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5]$ and x_i is the working interest on project i and $R_i(\omega)$ and $NPV_i(\omega)$ are the reserves and NPV of project i calculated stochastically according to the single project set of equations described in the previous chapter and using the inputs described at the beginning of this chapter.

7.5.1 Settings

MOGOL-XL was set to run over 150 generations. The number of replications per simulation was set to 2500. The mutation rate was set to 0.3 and the crossover rate was set to 0.4. The population size was set to 15.

7.5.2 Results

This section shows the results of the experiment 1. The running time of the algorithm was 16 hours and 47 minutes.

7.5.2.1 Assessment of the new "borders" of the decision space

Once an approximate Pareto set has been produced it is relevant to study if the optimisation has narrowed the decision space. Before the optimisation, the decision space was defined by the fact that the DMs may choose any working interest between 0 and 100% for any of the projects. After the optimisation the decision space may become narrower since the resulting set does not include suboptimal portfolios.

Table 7-9 shows the working interest combination that defines each of the 20 portfolios of the calculated approximate Pareto set. The bottom of the table shows the minimum and maximum values of the working interest for each of the projects. These results show that the decision space has been barely narrowed down as Project 1 do not take more than 90% working interest in any of the proposed portfolios.

The rest of the projects maintain their full range of decision alternatives with participation levels that vary from 0% to 100%. Hence, the decision space remains identical for these 4 projects after the optimisation. These results indicate that according to the set of objectives set for this optimisation there is not a project (or projects) that must be taken regardless of the portfolio chosen. This situation arises when a project shows 100% working interest for all of the portfolios generated in the Pareto set.

	x1	x2	x3	x4	x5
	%	%	%	%	%
Port 1	14.24%	4.58%	95.38%	95.48%	51.94%
Port 2	24.98%	3.87%	96.10%	96.18%	45.63%
Port 3	28.85%	3.51%	96.91%	97.21%	56.66%
Port 4	62.30%	1.39%	98.60%	98.60%	23.70%
Port 5	79.29%	0.65%	99.35%	99.36%	7.34%
Port 6	76.67%	3.93%	99.37%	96.08%	8.94%
Port 7	90.00%	0.00%	100.00%	100.00%	0.00%
Port 8	44.42%	0.00%	100.00%	100.00%	75.97%
Port 9	30.00%	0.00%	100.00%	100.00%	100.00%
Port 10	27.29%	12.59%	94.59%	98.33%	87.60%
Port 11	18.34%	38.87%	61.13%	100.00%	76.24%
Port 12	4.47%	87.62%	73.56%	99.77%	8.06%
Port 13	10.21%	57.10%	59.09%	67.90%	83.28%
Port 14	0.00%	100.00%	70.00%	100.00%	0.00%
Port 15	0.00%	100.00%	0.00%	100.00%	38.89%
Port 16	0.00%	73.33%	100.00%	0.00%	100.00%
Port 17	0.00%	100.00%	100.00%	0.00%	55.56%
Port 18	7.47%	86.99%	26.94%	16.07%	93.21%
Port 19	0.00%	100.00%	20.00%	0.00%	100.00%
Port 20	6.67%	100.00%	0.00%	0.00%	100.00%
Min	0.00%	0.00%	0.00%	0.00%	0.00%
Max	90.00%	100.00%	100.00%	100.00%	100.00%

Table 7-9 Composition of the approximate Pareto optimal portfolios

7.5.2.2 Assessment of the new "borders" of the objective space

After the multi-objective optimisation has been completed it is possible to assess what are the maximum and minimum values that the efficient portfolios may deliver in each one of the objectives under study. The main issue to keep in mind is that if there are trade-offs among the objectives, the portfolio that delivers the maximum in a given performance objective, is not necessarily the same portfolio that will deliver the maximum performance on another objective.

Table 7-10 shows that the calculated approximate Pareto set has a maximum E(NPV) potential of US$695.72 million and a minimum potential of US$117.92. In the same manner, the maximum probability of achieving a positive NPV is 97.50% and the minimum probability of achieving a positive NPV is 65.19%.

The same table shows that the maximum expected reserves is 198.93 million of barrels while the minimum potential expected reserves is 130.37 million barrels. Additionally the highest potential value for the 10th percentile of the reserves is 155.15 MMbbls and the minimum reserves P10 is 108.03 MMbbls.

Although the Capex is not an objective of the optimisation problem, it is relevant to verify that the resulting portfolio performance meets the relevant budget constraint of US$ 2.5 billion. Table 7-10 shows that the maximum possible Capex for the approximate Pareto set is the maximum allowed budget of US$ 2.5 billion and the minimum is US$ 1.85 billion.

	E(NPV)	P(NPV>0)	E(Reserves)	P10(Reserves)	CAPEX
	US$MM	%	MMbbls	MMbbls	US$MM
Port 1	541.52	97.50	130.37	108.03	1847.36
Port 2	564.29	96.91	136.93	114.58	1949.08
Port 3	583.91	96.70	148.99	124.77	2111.77
Port 4	643.41	94.78	159.72	125.88	2302.53
Port 5	673.93	92.88	165.83	124.98	2407.76
Port 6	661.44	92.85	166.79	126.75	2410.81
Port 7	695.72	91.49	171.67	126.73	2500.00
Port 8	642.61	95.90	176.84	145.89	2500.00
Port 9	625.81	95.94	178.47	148.32	2500.00
Port 10	601.20	95.48	179.74	152.46	2498.64
Port 11	517.17	94.49	183.75	155.15	2500.00
Port 12	493.16	91.65	185.12	140.95	2470.21
Port 13	417.17	90.85	187.90	153.48	2495.89
Port 14	476.67	89.69	188.76	139.54	2500.00
Port 15	346.34	83.16	192.06	140.54	2500.00
Port 16	311.89	81.63	193.52	150.72	2500.00
Port 17	296.33	78.82	194.98	144.37	2500.00
Port 18	214.19	76.42	195.23	150.41	2494.83
Port 19	147.39	68.34	198.75	147.95	2500.00
Port 20	117.92	65.19	198.93	148.03	2500.00
Min	117.92	65.19	130.37	108.03	1847.36
Max	695.72	97.50	198.93	155.15	2500.00

Table 7-10 Performance of the approximate Pareto set in the 4 objectives under study and the Capex

7.5.2.3 Pareto set

The basic premise to perform a multi-objective optimisation is the presence of tradeoffs between the objectives to be optimised. This section aims to assess the presence of such trade-offs in the calculated approximate Pareto set.

It is relevant to state that the presence or absence of tradeoffs among the objectives under study is inherent to this particular work and these results cannot be generalized to any exploration and production portfolio. However, the idea is to establish a methodology to unveil these tradeoffs to the DM. Figure 7-2 shows the resulting approximate Pareto set showing the performance of each one of the 20 portfolios calculated for each one of the objectives set plus their total costs. The left vertical axis shows the performance of the two objectives calculated over the attribute reserves and the probability of achieving a positive NPV. The right vertical axis shows the performance of the E(NPV). The lower horizontal axis shows the name of the portfolio and the upper horizontal axis shows the total cost (Capex) of the portfolios.

The portfolios where sorted and named based in the expected reserves attribute. In this manner portfolio 1 and portfolio 20 have the minimum and maximum performance over the objective E(Reserves) respectively. Note that none of the portfolios is dominated by any other of the portfolios present in the graph. Figure 7-3 shows the composition of each one of the portfolios in terms of working interest. In this figure the height of the stacked bars represent the working interest proposed for each project. Analysing both figures jointly may assist the DM in tracing the impact of the presence of a project in the overall performance of a given portfolio.

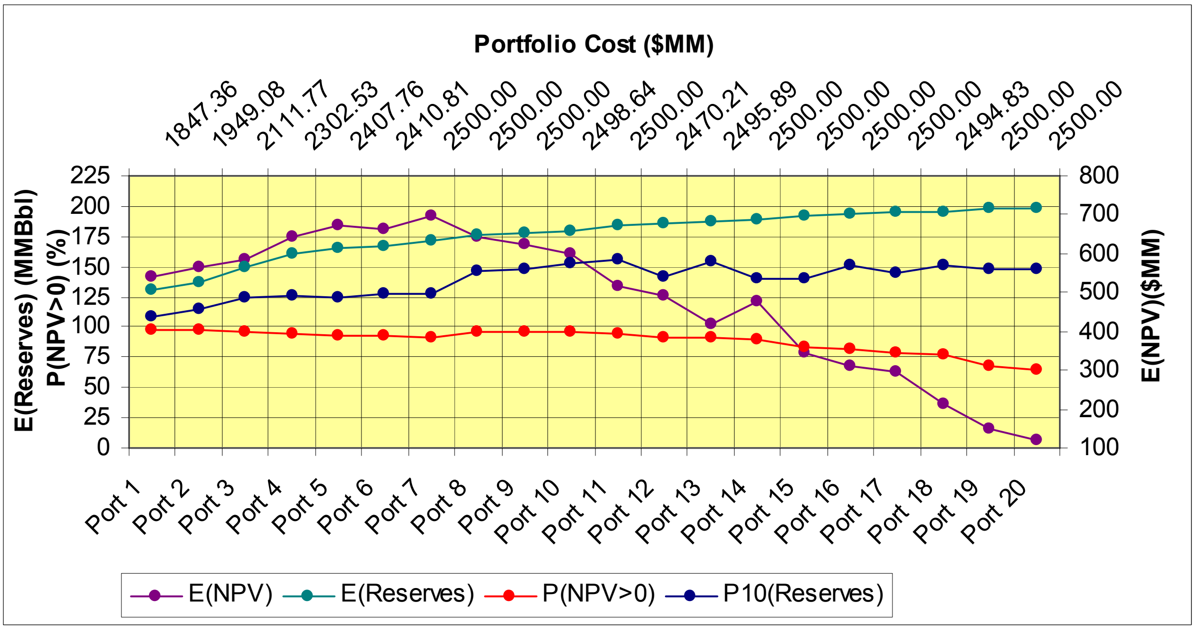
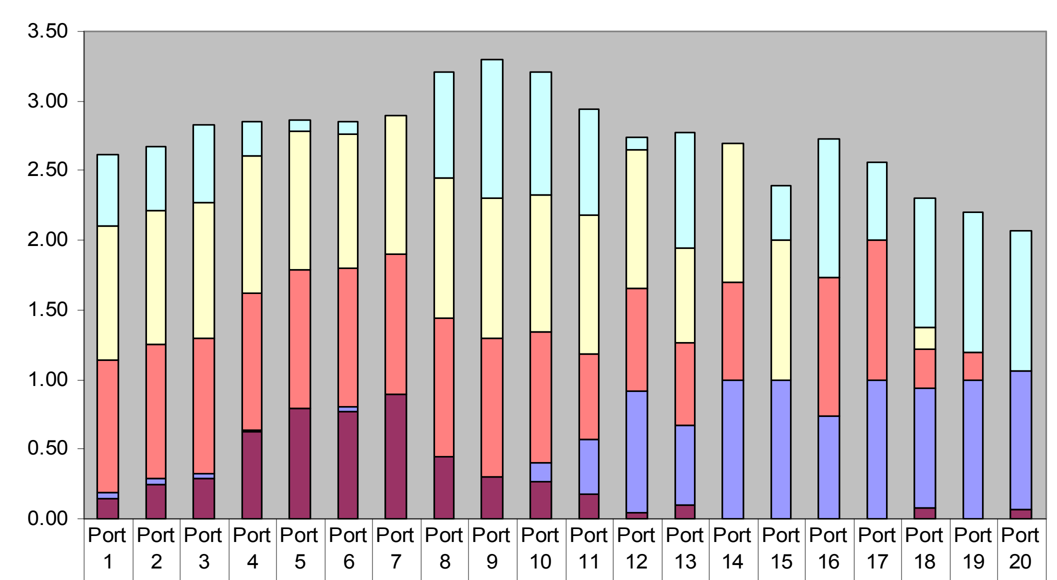


Figure 7-2 Performance of the approximate Pareto set over the objectives E(Reserves), P10(Reserves),P(NPV>0) and E(NPV)



	Port 1	Port 2	Port 3	Port 4	Port 5	Port 6	Port 7	Port 8	Port 9	Port 10	Port 11	Port 12	Port 13	Port 14	Port 15	Port 16	Port 17	Port 18	Port 19	Port 20
x_5	0.52	0.46	0.57	0.24	0.07	0.09	0.00	0.76	1.00	0.88	0.76	0.08	0.83	0.00	0.39	1.00	0.56	0.93	1.00	1.00
x_4	0.95	0.96	0.97	0.99	0.99	0.96	1.00	1.00	1.00	0.98	1.00	1.00	0.68	1.00	1.00	0.00	0.00	0.16	0.00	0.00
x_3	0.95	0.96	0.97	0.99	0.99	0.99	1.00	1.00	1.00	0.95	0.61	0.74	0.59	0.70	0.00	1.00	1.00	0.27	0.20	0.00
x_2	0.05	0.04	0.04	0.01	0.01	0.04	0.00	0.00	0.00	0.13	0.39	0.88	0.57	1.00	1.00	0.73	1.00	0.87	1.00	1.00
x_1	0.14	0.25	0.29	0.62	0.79	0.77	0.90	0.44	0.30	0.27	0.18	0.04	0.10	0.00	0.00	0.00	0.00	0.07	0.00	0.07

Figure 7-3 Composition of the approximate Pareto set of portfolios

7.5.3 Result analysis

7.5.3.1 Trade-off screening

Although it is known that the convergence of the objectives set over the NPV attribute has a considerable range of variability, it is definitely possible to detect trends in the Figure 7-2. A visual screening of figure 7-2 allows inferring the following tradeoffs:

- The expected reserves appear to be negatively correlated to the probability of achieving a positive NPV.
- The 10^{th} percentile of the reserves tends to increase as the expected reserves increase.
- The expected NPV increases with the expected reserves from portfolio 1 to portfolio 7. Then the expected NPV decreases with increasing expected reserves from portfolio 8 to portfolio 20.
- The probability of achieving a positive NPV decreases as the expected NPV increases from portfolio 1 to portfolio 7 and then both objectives decrease together from portfolio 8 to portfolio 20.

The visual screening of Figure 7-2 allows separating the Pareto set into two regions, one region going from portfolio 1 to portfolio 7 and a second region going from portfolio 8 to portfolio 20. Note that the first region shows an increasing portfolio cost until the maximum feasible cost is reached. Hence, the inflexion point seems to be related with reaching the maximum capital available since the capital remains close to its maximum from that point onwards. Additionally, it is possible to note from Figure 7-3 that this area shows an increasing presence of project 1, a decreasing presence of project 5 and a steady presence of projects 2, 3 and 4 where the participation in project 2 is marginal (close to zero) and projects 3 and 4 are almost fully funded for all portfolios in this area.

From all of the above it is possible to conclude that the capital constraint is the source of the negative correlation observed from portfolio 8 to portfolio 20 between the E(NPV) and the objectives based on the reserves attribute. Additionally, the capital constraint also seem to be the source of positive correlation between the E(NPV) and the P(NPV>0) from portfolio 8 to portfolio 20.

The presence of project 1 decreases from portfolio 7 onwards and becomes marginal (<10%) from portfolio 14 onwards. Hence, it is possible to infer that the presence of project 1 is related to the best performing portfolios in terms of the E(NPV) objective. On the contrary, the presence of project 2 seems to be related with the portfolios that best perform in terms of the expected value of the reserves but also with the ones that perform worst in terms of E(NPV) and the probability of achieving a positive NPV.

It is also possible to infer that the presence of projects 1 and 2 negatively impacts the chances of achieving a positive NPV since the portfolio that performs best in the P(NPV>0) has low participation of both projects and is also the cheapest one. On the other hand it is possible to note that the portfolios with higher expected reserves and P10 of the reserves are the ones with least probabilities of achieving a positive NPV, this area of the Pareto set has a stronger presence of projects 2 and 5. Particularly, it is possible to note that the only portfolio with a slightly better performance on the E(NPV) area is portfolio 14, the only one that lacks the presence of project 5.

Figure 7-3 also shows that projects 3 and 4 have high participation levels for most of the Pareto optimal portfolios and tend to disappear in the portfolios with higher reserves and lower probabilities of achieving a positive NPV.

7.5.3.2 Preference articulation

The previous discussion showed that it is possible to quickly gain insight about the trade-offs among the objectives of a portfolio and about the impact of specific projects on these objectives by plotting the results of the calculated approximate Pareto set. However, let us assume that with the same set of projects a DM decides to use single objective optimisation and sets a simple portfolio problem in which the DM wishes to maximise the E(NPV) of the portfolio whilst keeping the E(R) higher than 180MMbbls.

In this case, only the portfolios from portfolio 11 to portfolio 20 would be in the feasible region and according to the maximisation criteria, portfolio 11 would be selected as it has the highest E(NPV). Yet, it is possible to note that portfolio 10 has a similar performance than portfolio 11 in the objectives P(NPV>0) and P10(R) but it barely misses the constraint of 180MMbbls (see table 7-10) with a performance in this objective of 179.74MMbbls. However, portfolio 10 delivers $84 million more than portfolio 11 on the E(NPV) objective.

Would the DM be willing to trade 0.26 MMbbls in the E(R) objective in order to gain $84 million more on the E(NPV) objective? The answer would definitely depend on the preferences of the DM. However, regardless of the choice, the decision would have more insight if the DM is aware of the portfolio 10 alternative. However, the DM would probably have failed to detect the presence of this sort of trade-off with the use of single objective methods.

7.6 Experiment 2: Multiple objectives over a single attribute

In this section the proposed portfolio optimisation model is again applied here to the five projects shown in the previous section. The same capital constraint of US$2.5 million will be used. However, it is assumed that in this case the DMs are just interested in the NPV attribute. Moreover, the DMs are not only interested in maximising the expected value of the NPV and the probability of achieving a positive value for this attribute. In this case, the DMs also want to maximise the 10^{th} percentile of the NPV attribute in other to minimise the chances of a negative outcome of large magnitude. Additionally, the DM wants to maximise the 90^{th} percentile of the NPV and therefore enhance the chances of an upside outcome of large magnitude. This statement can be summarised mathematically as follows:

Maximise:

$$[E(NPV_{Port}(\mathbf{X},\omega)), P(NPV_{Port}(\mathbf{X},\omega) > 0), P10(NPV_{Port}(\mathbf{X},\omega)), P90(NPV_{Port}(\mathbf{X},\omega))]$$

Subject to:

$$CapEx_{Port}(\mathbf{X}) = \sum_{i=1}^{5} x_i CapEx_i \leq US\$MM\,2500$$

$$NPV_{Port}(\mathbf{X},\omega) = \sum_{i=1}^{5} x_i NPV_i(\omega)$$

And, $0 \leq \mathbf{X} \leq 1$

Where $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5]$ and x_i is the working interest on project i.

7.6.1 Settings

MOGOL-XL was set to produce 100 generations of frontiers. The number of iterations per simulation was set to 2500. The mutation rate was set to 0.3 and the crossover rate was set to 0.4. The population size was set to 15.

7.6.2 Results

This section shows the results of the experiment 1. The running time of the algorithm was approximately 12 hours and 23 minutes.

7.6.2.1 Assessment of the new "borders" of the decision space

Table 7-11 shows the working interest combination that defines each of the 20 portfolios of the calculated approximate Pareto set. The bottom of the table shows the minimum and maximum values presented in the working interest of each of the projects. These results show that the decision space has been considerably narrowed down.

The participation of project 3 barely varies with a minimum of 98.28% percent and a maximum of 100%. Hence, it is possible to consider that fully funding this project is a must regardless of what other projects are selected. On the contrary, the maximum participation level found for project 2 is less than 1% and hence it is possible to conclude that this project should not enter the portfolio at all.

Project 4 shows a minimum 76.92% and a maximum of 100%. Similarly, although the range of alternatives is higher it is possible to conclude that a high participation on this project is also necessary. Additionally, Project 1 has a minimum of 8.48% and a maximum of 100%. The participation of project 5 is not affected by the optimisation and hence varies from 0% to 100%.

	x1	x2	x3	x4	x5
	%	%	%	%	%
Port 1	100.00	0.00	100.00	76.92	0.00
Port 2	90.00	0.00	100.00	100.00	0.00
Port 3	83.24	0.04	99.34	99.94	11.02
Port 4	80.98	0.03	99.02	99.91	14.89
Port 5	83.62	0.03	99.36	99.41	10.77
Port 6	72.17	0.17	99.29	99.98	28.45
Port 7	71.20	0.16	99.09	99.97	30.55
Port 8	64.54	0.06	98.28	97.76	44.42
Port 9	57.32	0.09	99.24	99.95	42.55
Port 10	48.13	0.01	99.78	99.98	69.70
Port 11	46.23	0.06	99.51	99.97	65.23
Port 12	58.87	0.05	99.31	99.94	38.91
Port 13	33.48	0.00	99.96	100.00	94.19
Port 14	53.68	0.10	99.23	99.96	46.98
Port 15	30.00	0.00	100.00	100.00	100.00
Port 16	29.70	0.03	99.68	99.98	60.33
Port 17	8.48	0.03	99.95	100.00	75.91
Port 18	13.67	0.51	98.94	99.88	46.68
Port 19	18.96	0.56	98.77	99.86	38.96
Port 20	9.10	0.65	98.73	99.85	42.79
Min	8.48	0.00	98.28	76.92	0.00
Max	100.00	0.65	100.00	100.00	100.00

Table 7-11 Composition of the approximate Pareto optimal portfolios

7.6.2.2 Assessment of the new "borders" of the objective space

Table 7-12 shows that the calculated approximate Pareto set has a maximum E(NPV) potential of US$715.22 million and a minimum potential of US$555.57 million. As expected, the maximum probability of achieving a positive NPV is 97.87% and the minimum probability of achieving a positive NPV is 89.30%.

The same table shows that the minimum P10(NPV) achieved is -US$21.12 million and a maximum P10(NPV) of US$ 188.58 million. On the other hand the results show a minimum P90(NPV) of -US$ 21.12 and maximum P90(NPV) of US$188.58 million .

The maximum Capex meets the budget constraint of US$ 2.5 billion and the minimum CAPEX shown is US$ 1.673 billion.

	E(NPV)	P(NPV>0)	P10(NPV)	P90(NPV)	CAPEX
	US$MM	%	MMbbls	MMbbls	US$MM
Port 1	679.58	89.30	-21.12	1395.85	2500.00
Port 2	710.04	91.88	50.85	1378.56	2500.00
Port 3	703.20	92.57	70.16	1369.36	2494.58
Port 4	714.11	93.23	95.94	1354.54	2493.66
Port 5	715.22	93.34	84.77	1368.69	2494.70
Port 6	691.71	93.90	101.85	1316.54	2487.51
Port 7	682.40	94.10	119.98	1286.01	2490.59
Port 8	674.03	94.21	110.66	1264.75	2495.54
Port 9	664.96	94.52	125.97	1225.72	2389.99
Port 10	669.89	95.03	143.96	1225.31	2498.15
Port 11	660.65	95.28	153.31	1215.27	2428.74
Port 12	667.96	95.33	134.74	1223.95	2380.11
Port 13	645.44	95.45	158.51	1168.35	2499.65
Port 14	665.74	96.02	150.14	1211.31	2375.29
Port 15	651.86	96.25	182.18	1149.20	2500.00
Port 16	629.72	97.22	188.58	1086.34	2137.19
Port 17	580.08	97.27	184.41	1007.42	1960.54
Port 18	567.26	97.37	172.65	973.96	1776.83
Port 19	575.54	97.38	162.57	988.46	1786.38
Port 20	555.57	97.87	168.04	971.82	1673.99
Min	555.57	89.30	-21.12	971.82	1673.99
Max	715.22	97.87	188.58	1395.85	2500.00

Table 7-12 Performance of the approximate Pareto set in the 4 objectives under study and the Capex

7.6.2.3 Pareto set

Figure 7-4 shows the performance of the Pareto set in terms of the optimised objectives. In this figure the left vertical axis shows the performance of the objectives E(NPV), P10(NPV), P90(NPV) and the right vertical axis show the performance of the P(NPV>0) objective. The upper horizontal axis shows the total Capex required to fund the proposed portfolios while the lower horizontal axis shows the name of the portfolios. The portfolios have been sorted and named based in the probability of achieving a positive NPV objective. Hence, portfolio 1 has the minimum probability of achieving a positive NPV while portfolio 20 has the maximum probability of achieving a positive NPV. Additionally, Figure 7-5 shows the composition of the portfolios of the Pareto set. In this figure, the height of the stacked bars represents the working interest proposed for each project.

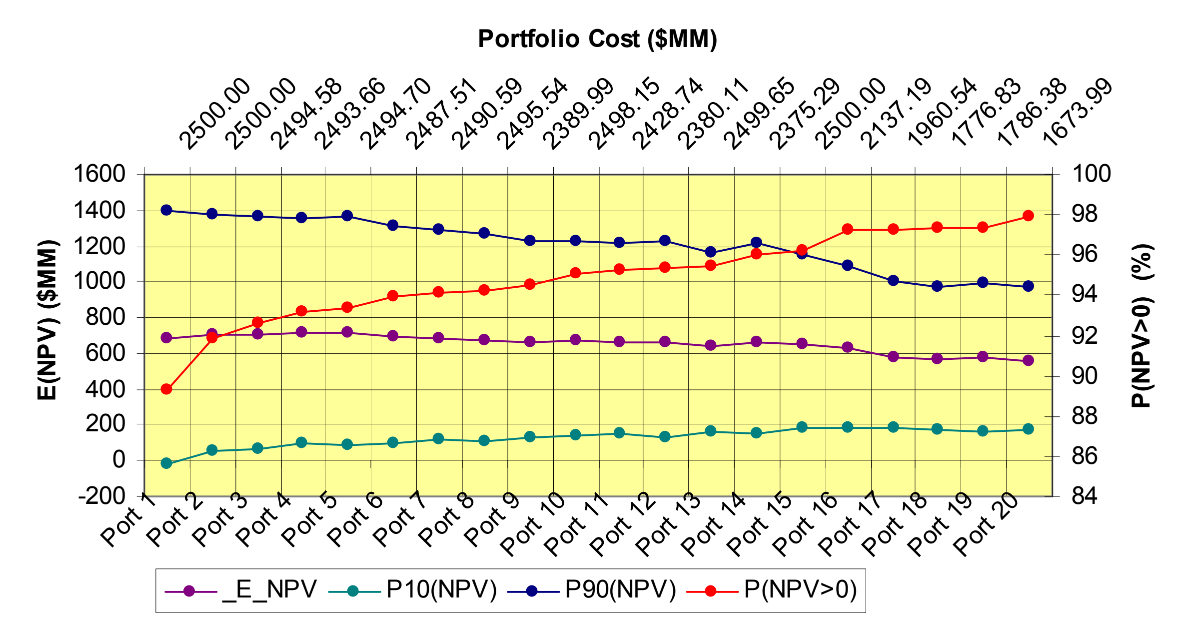
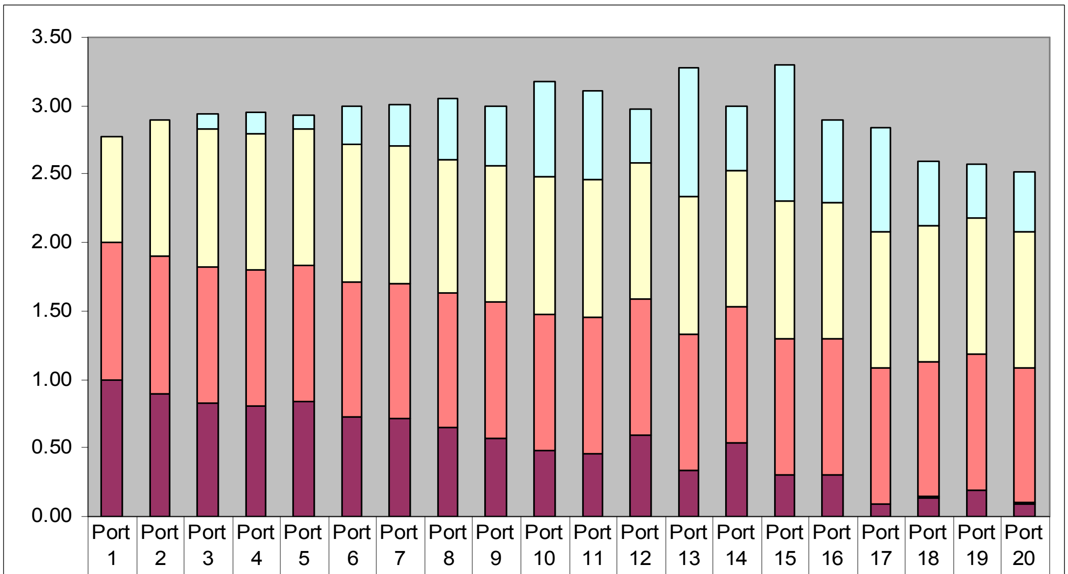


Figure 7-4 Performance of the approximate Pareto set over the objectives E(NPV), P10(NPV),P(NPV>0) and P90(NPV)



	Port 1	Port 2	Port 3	Port 4	Port 5	Port 6	Port 7	Port 8	Port 9	Port 10	Port 11	Port 12	Port 13	Port 14	Port 15	Port 16	Port 17	Port 18	Port 19	Port 20
x_5	0.00	0.00	0.11	0.15	0.11	0.28	0.31	0.44	0.43	0.70	0.65	0.39	0.94	0.47	1.00	0.60	0.76	0.47	0.39	0.43
x_4	0.77	1.00	1.00	1.00	0.99	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
x_3	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.98	0.99	1.00	1.00	0.99	1.00	0.99	1.00	1.00	1.00	0.99	0.99	0.99
x_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01
x_1	1.00	0.90	0.83	0.81	0.84	0.72	0.71	0.65	0.57	0.48	0.46	0.59	0.33	0.54	0.30	0.30	0.08	0.14	0.19	0.09

Figure 7-5 Composition of the approximate Pareto set of portfolios

7.6.3 Result analysis

7.6.3.1 Trade-off screening

In a similar manner to the previous experiment, although it is known that the convergence of the objectives set over the NPV attribute has a considerable variability it is possible to detect trends in the Figure 7-4. This figure allows inferring the following tradeoffs:

- The P10(NPV) tends to increase as the P90(NPV) tends to decrease.
- The probability of achieving a positive NPV increases as the expected NPV decreases.
- The expected NPV tends to decrease as the P10 increases and the P90 decreases.

The previous observations suggest that the volatility of the NPV attribute decreases as the probabilities of achieving a positive NPV increases. This observation is perfectly aligned with mean-variance optimisation approach since this method predicts that the higher the E(NPV) the higher the volatility of the of the NPV. It is also possible to note that the absolute distance of the P10(NPV) and P90(NPV) objectives to the E(NPV) is quite similar in each one of the portfolios. This allows inferring that the shapes of the NPV PDFs tend to be symmetrical. And hence it is not necessary to set the P10(NPV) and the P90(NPV) as additional objectives since the idea behind this is that the distributions may be log-normally distributed.

Figure 7-5 shows that projects 3 and 4 have high participation levels for all the proposed portfolios. This figure also shows that the main source of variability in the composition of the portfolios is the presence of project 1 and project 5. It is possible to note as well that the presence of project one increases the volatility of the portfolios whilst the presence of project 5 diminishes it.

7.6.3.2 Preference articulation

The results in Figure 7-4 seem to be equivalent to the results generated by a mean-variance portfolio optimisation. It is possible to infer that these results are a consequence of the small asymmetry shown in the distributions of the PDFs of the NPVs of the project proposals.

However, it is possible to state that the use of the variance or the standard deviation as a risk/uncertainty metric requires more statistical training from the DMs to be interpreted than the probabilities and percentiles shown in Figure 7-4. In chapter 3 it was shown that most investors are not only concerned with the probabilities of achieving a desired target but also concerned about the magnitude of a disastrous outcome. In this manner, the method proposed here not only directly optimises the objectives of interest but also provides an easier way to communicate the results to DMs with little statistical training.

7.7 Summing up

This chapter described the implementation of the proposed portfolio optimisation model to a group of five projects. The inputs and outputs of each project proposal were presented. A brief study of the convergence of the stochastic objectives used in the experiments was shown and concluded that the objectives set over the NPV show a considerable variability for simulations of 2500 iterations. The need to keep the number of iterations low to avoid unpractical solution times was discussed.

The results shown in experiment 1 described the presence of complex tradeoffs among objectives set over multiple attributes of an E&P project portfolio. It was also shown that being aware of these tradeoffs may assist DMs to make better decisions.

The results shown in experiment 2 described how stochastic multi-objective optimisation can be used to optimise several objectives over a single attribute. The potential use of this methodology to better characterise and communicate the risks associated with the efficient portfolios was addressed.

By comparing the results of both experiments it is possible to note that the composition of the Pareto sets produced by the two experiments varies considerably.

8 CONCLUSION AND DISCUSSION

8.1 Introduction

This thesis has described the development of a framework for the selection of E&P project portfolios with multiple objectives set over multiple attributes under uncertain conditions whilst accounting for inter-project and intra-project correlations. In this final chapter the main results are recapitulated and the possible extensions of this research are discussed.

8.2 Results

8.2.1 Multi-objective E&P project portfolio optimisation model

The proposed model is a decision support aimed to aid E&P DMs to select project portfolios in situations where there are multiple decision attributes and the future performance of these attributes is uncertain. The model is based on ideas extracted from the fields of multi-objective optimisation, stochastic optimisation and decision and risk analysis.

The model differs from those presented in the E&P literature by producing a Pareto set involving more than two objectives and hence requiring DMs to make a decision after they have been exposed to a representative set of the tradeoffs among these objectives. In other words, the model extends the risk-return method proposed by Markowitz (1952) to multiple attributes. Additionally, it was shown that the method here proposed allows setting multiple objectives over each one of the attributes under consideration. Moreover, the objectives set over these attributes do not need to be defined by an equation since the method is based on a simulation-optimisation approach.

The method is mostly based in the assumption that if a DM has been exposed to the full range of possible tradeoffs present among the objectives under consideration she will gain more insight about the problem and hence would make more informed and better decisions. Additionally, the fact that the method uses posterior articulation of preferences guarantees that feasible solutions will be found.

The model is designed to honour the presence of inter and intra-project correlations through the integration of the stochastic characterisation of each project in terms of the various attributes of interest with the optimisation process. In addition, this integrated approach promotes consistency in the characterisation of each one of the project proposals under study. On the other hand, the model provides the analyst the flexibility to add changes to the model(s) that characterises each one of the projects under study since projects are fully modelled in Excel™ and Crystal Ball™.

The main flaw of the model proposed is its inability to handle non-linear constraints. This limitation does not allow modelling the inclusion or exclusion of projects based on the presence of other projects. In the same manner it is not possible to model mutually exclusive projects. This is a main limitation to include the time to start projects as an additional decision variable. Without the linear constraint limitation it would be possible to model the same project starting at different times and then set it as mutually exclusive from itself starting at different times.

Other main limitation is the running time of the Monte Carlo simulations. With shorter simulation times it would be possible to run more iterations per simulation and hence get better convergence of the objectives as well as the possibility to include a larger set of project proposals. A final limitation of the model is the visualisation of the Pareto set with an increasing number of objectives.

8.2.2 Application of the portfolio optimisation model

The Pareto sets calculated in experiments 1 and 2 demonstrated the presence of complex trade-offs among the objectives to be optimised. Particularly, the level of complexity observed among two key objectives as the E(R) and the E(NPV) is such that it is highly improbable that DMs would have an adequate intuitive feel for this trade-off when setting goals over these objectives without any other source of information. Consequently, this fact would difficult the calculation of feasible solutions.

Hence, the use of multi-objective optimisation with posterior articulation of preferences seems to be a promising way to help DMs to gain insight about the maximum potential of their portfolios without having to rely in their intuition. However, the method here proposed does not necessarily have to be seen as a substitute to the use of goals since it is widely known that goals are a good way to monitor performance. Instead the method here proposed can be seen as a complement to the use of goals. In this manner goals could be set with more insight and without the risk of setting combinations of goals that are unfeasible to achieve as a result of tradeoffs among them.

8.3 Further research and model development

This thesis raises several interesting topics of future research. An obvious topic to improve the research here presented would be to shorten the solution time of the Monte Carlo simulations so that more iterations could be used per simulation in order to obtain more accurate solutions. This could be achieved by coding the equations that describe the single project models, the portfolio model and the Monte Carlo simulation capability in the same language used by MOGOL (JAVA™). In this manner some flexibility to model the projects would be lost but the results would definitely be obtained faster since it would not be necessary to transfer data among applications. This would allow testing the algorithm with a larger number of projects that would better resemble the decision situation faced by E&P DMs in the real world where it could be necessary to screen hundreds of project proposals.

Another line of research would be to expand the model here presented to include the time dimension into the portfolio by developing an algorithm capable of not only optimising the working interest of the projects but also their starting times. As stated before, this research should probably concentrate in sorting out a way to include non-linear constraints in the problem. In this manner the same project could be included as a different proposal starting in a different year and then this project could be defined as mutually exclusive from itself starting at different times.

Another possible topic of research would be to generate Pareto optimal sets of the more common objectives found in the literature for as many and as diverse as possible combinations of E&P projects in order to try to identify "typical" tradeoffs for the objectives of interest. For example, just as it is widely known in the literature the nature of the shape that describes the trade-off between the mean and the variance of the NPV, it would be a good source of insight for DMs to know the typical trade-off between the E(R) and the E(NPV). However, it would be necessary to test first if there is such a thing as a "typical" trade-off for these two objectives.

Other topic of research would be to test the value of characterising the risk of an investment using multiple objectives as shown in experiment 2. In this manner the results of this approach could be compared with the results obtained for similar set of assets using more advanced risk measures than the ones discussed in this thesis (i.e., coherent risk metrics, value at risk).

Another topic of research that would strongly validate the value of the approach presented in this thesis would be to investigate the difference in the quality of decisions made with prior articulation of preferences and decisions made with posterior articulation of preferences. In this manner a group of DMs should be asked to make an investment decision seeing all the possible alternatives (portfolios) and their forecasted performance on multiple objectives first. Secondly the same group should be asked to firstly express their preferences via a multi-attribute utility function or via the setting of goals on the relevant objectives and then find a solution that meets their expectations (if such a solution exists). Then the answers from the two cases could be analysed to conclude which method promotes better decisions.

Another topic of research regarding the application of the multi-objective optimisation techniques to the E&P industry could be the application of these techniques to the design of petroleum fiscal regimes, especially production sharing contracts (PSC). It is widely known that these contracts are usually highly complex systems were governments seek to maximise their profit while assuring attractive benefits to the oil companies wishing to invest in the country. This obvious trade-off is a function of the various variables that characterise the PSC (i.e., bonuses, profit share splits, cost recovery limits). Hence, these variables could be set as the decision variables and the government profits and the contractor profits could be set as objectives. In this manner a Pareto set of potential "contracts" could be screened to select the type of contract that best suits the preferences of both parties.

9 APPENDIX A: SPE RESERVES AND RESOURCES DEFINITIONS SPE (2000)

DISCOVERED PETROLEUM-INITIALLY-IN-PLACE: "Discovered Petroleum-initially-in-place is that quantity of petroleum that is estimated, on a given date, to be contained in known accumulations, plus those quantities already produced. Discovered Petroleum-initially-in-place may be subdivided into Commercial and Sub-commercial categories, with the estimated potentially recoverable portion being classified as Reserves and Contingent Resources respectively, as defined below."

ESTIMATED ULTIMATE RECOVERY: "Estimated Ultimate Recovery (EUR) is not a resource category as such, but a term that may be applied to an individual accumulation of any status/maturity (discovered or undiscovered). Estimated Ultimate Recovery is defined as those quantities of petroleum which are estimated, on a given date, to be potentially recoverable from an accumulation, plus those quantities already produced from there."

RESERVES: "Reserves are defined as those quantities of petroleum that are anticipated to be commercially recovered from known accumulations from a given date forward."

PROBABILISTIC MODEL: A method to estimate reserves is called probabilistic when "the known geological, engineering, and economic data are used to generate a range of estimates and their associated probabilities. Identifying reserves as proved, probable, and possible has been the most frequent classification method and gives an indication of the probability of recovery."

Proved Reserves

Proved reserves are those quantities of petroleum which, by analysis of geological and engineering data, can be estimated with reasonable certainty to be commercially recoverable, from a given date forward, from known reservoirs and under current economic conditions, operating methods, and government regulations. Proved reserves can be categorized as developed or undeveloped.

If deterministic methods are used, the term reasonable certainty is intended to express a high degree of confidence that the quantities will be recovered. If probabilistic methods are used, there should be at least a 90% probability that the quantities actually recovered will equal or exceed the estimate.

Probable Reserves

Probable reserves are those unproved reserves which analysis of geological and engineering data suggests are more likely than not to be recoverable. In this context, when probabilistic methods are used, there should be at least a 50% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable reserves.

Possible Reserves

Possible reserves are those unproved reserves which analysis of geological and engineering data suggests are less likely to be recoverable than probable reserves. In this context, when probabilistic methods are used, there should be at least a 10% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable plus possible reserves.

10 APPENDIX B: VOLUMETRIC EQUATION (ORIGINAL OIL IN PLACE)

The volumetric equation to calculate the OOIP is:

$$N = 7758 Ah\phi(1 - S_w) / B_0$$

Where:

$$7758 = \frac{43{,}560\text{ft}^2 / Ac}{5.614\text{ft}^3 / bbl}$$

N = OOIP, STB

A= area, Ac

H= average thickness, ft (oil interval)

ϕ= average porosity, fraction

S_w= average water saturation, fraction

B_0= average oil formation volume factor, RB/STB

11 APPENDIX C: PRODUCTION PROFILES, CASH FLOW PROFILES, NPV DISTRIBUTIONS AND RESERVES DISTRIBUTIONS OF PROJECTS 2, 3, 4 AND 5

This appendix shows the full PDFs of the attributes reserves and NPV and the P10, P50 and P90 cash flow and production profiles for each one of the project proposals.

It is important to note that it is common practice in the industry to call P10, P50 and P90 net cash flow and production profiles to profiles that correspond to the production of the P10, P50 and P90 of the EUR. This is not the case shown here where what is meant is that the P10, P50 and P90 profiles show that the attribute being plotted has 10% chance, 50% chance and 90% chance of being less than the value of the profile for that year.

It is possible to note from the NPV distributions showed below that for that most of these distributions are symmetrical or sightly skewed to the right. On the other hand, the PDFs of the reserves attribute show a log-normal behaviour.

11.1 Project 1

Production profile:

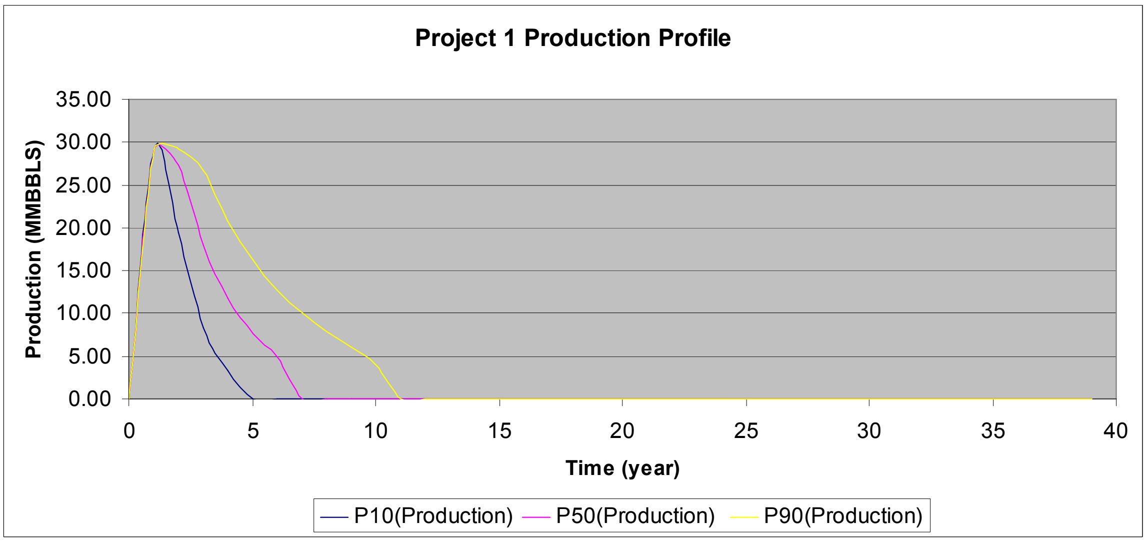


Figure 11-1 P10, P50 and P90 production of project 1

After tax cash flow profile:

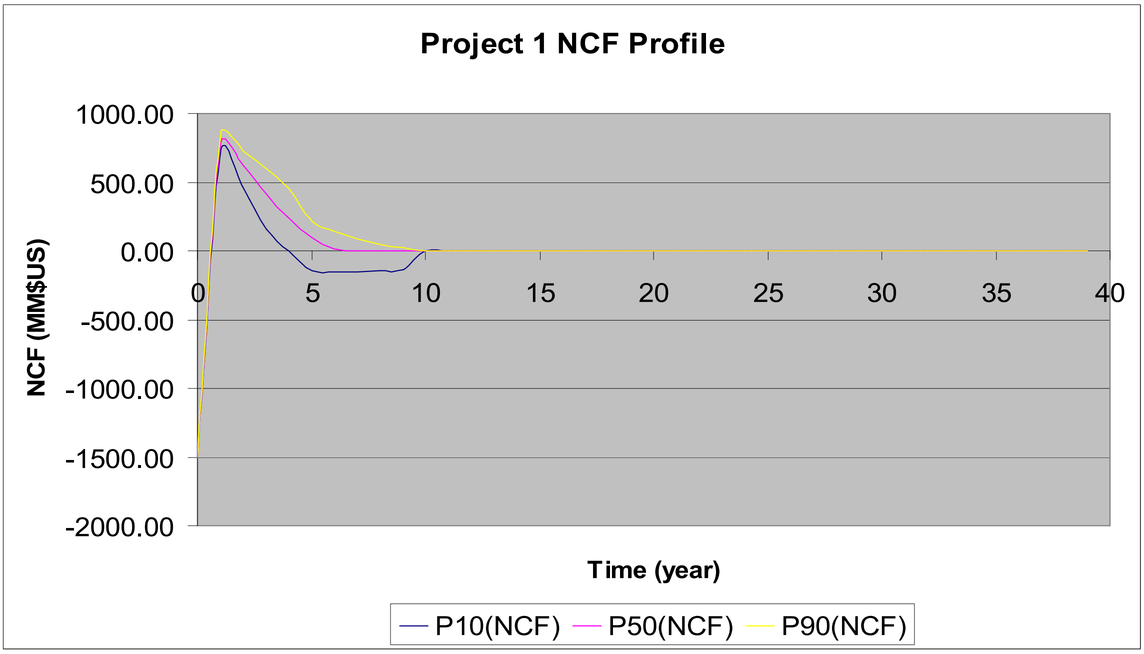


Figure 11-2 P10, P50 and P90 NCF of project 1

NPV distribution:

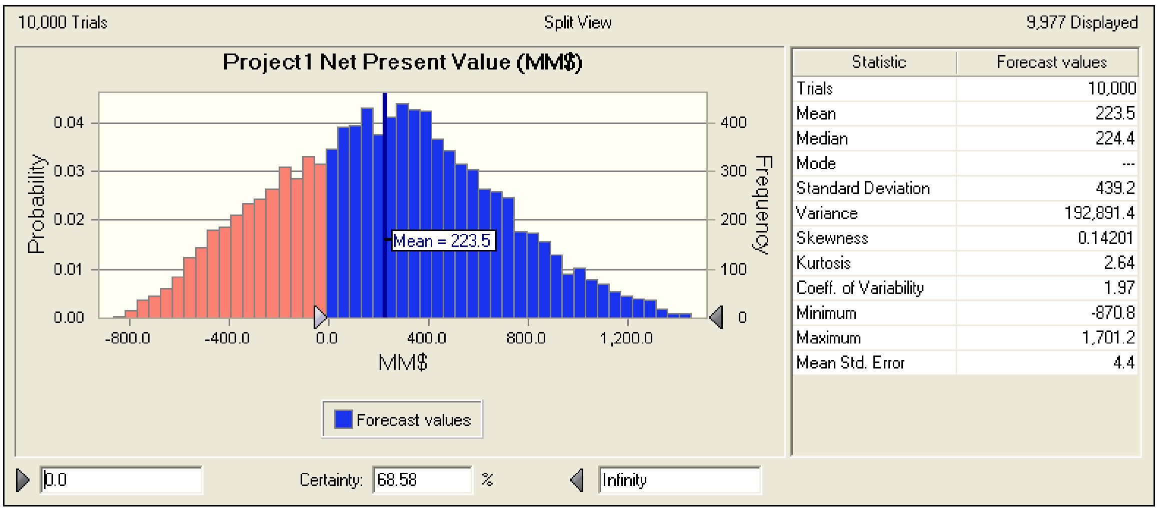


Figure 11-3 NPV distribution of project 1

Reserves distribution:

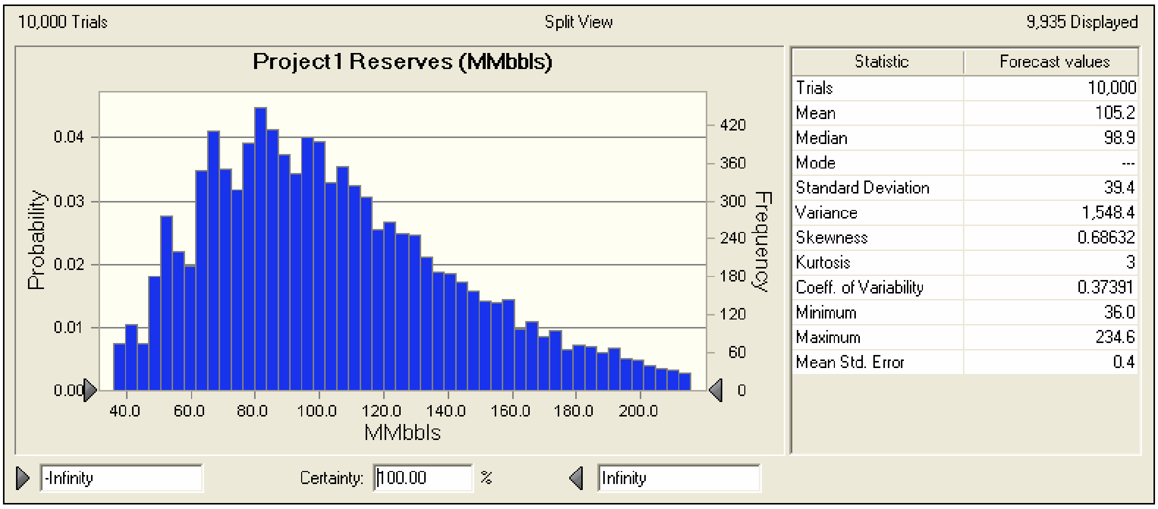


Figure 11-4 Reserves distribution of project 1

11.2 Project 2

Production profile:

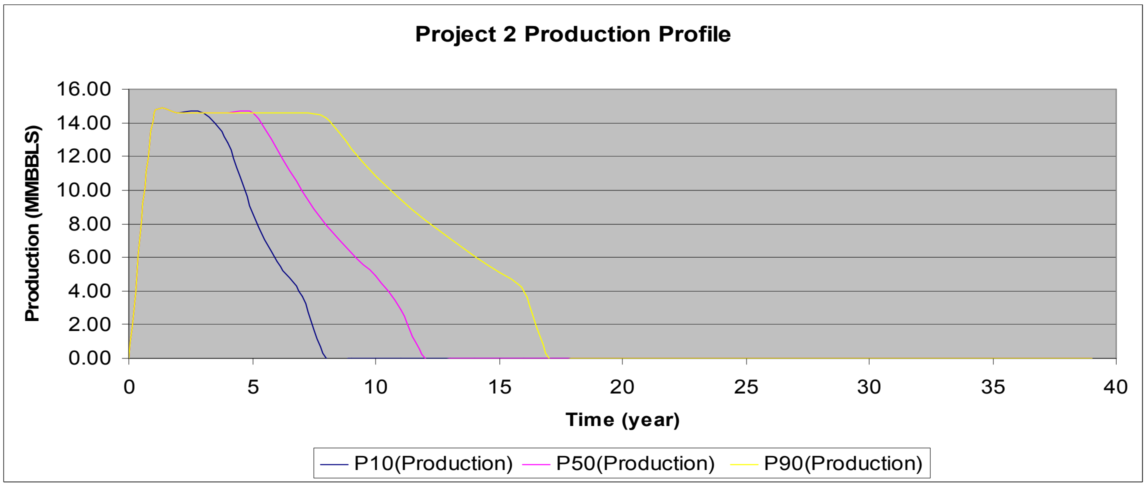


Figure 11-5 P10, P50 and P90 production of project 2

After tax cash flow profile:

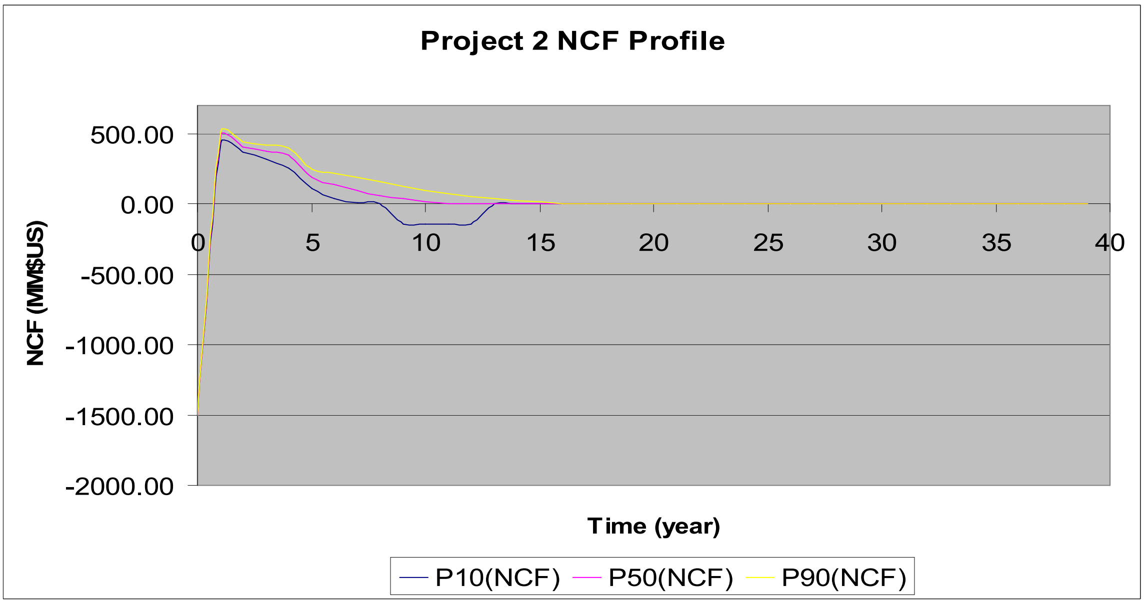


Figure 11-6 P10, P50 and P90 NCF of project 2

NPV distribution:

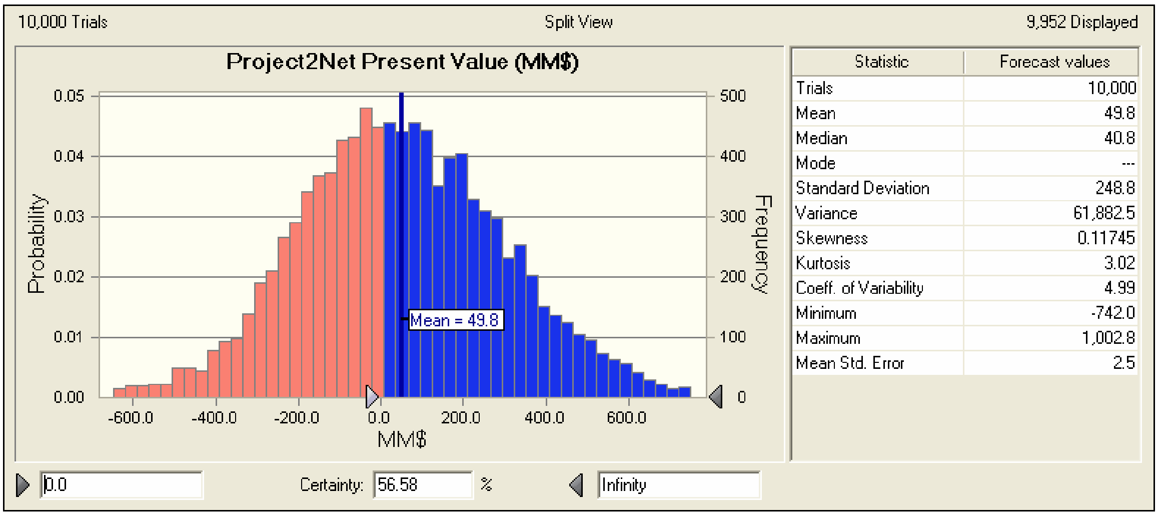


Figure 11-7 NPV distribution of project 2

Reserves distribution:

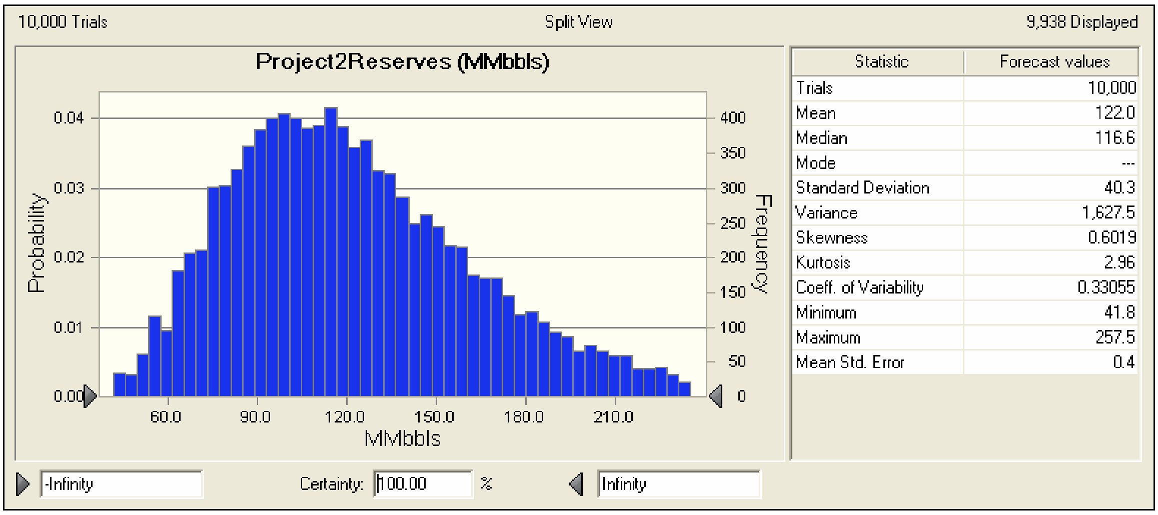


Figure 11-8 Reserves distribution of project 2

11.3 Project 3

Production profile:

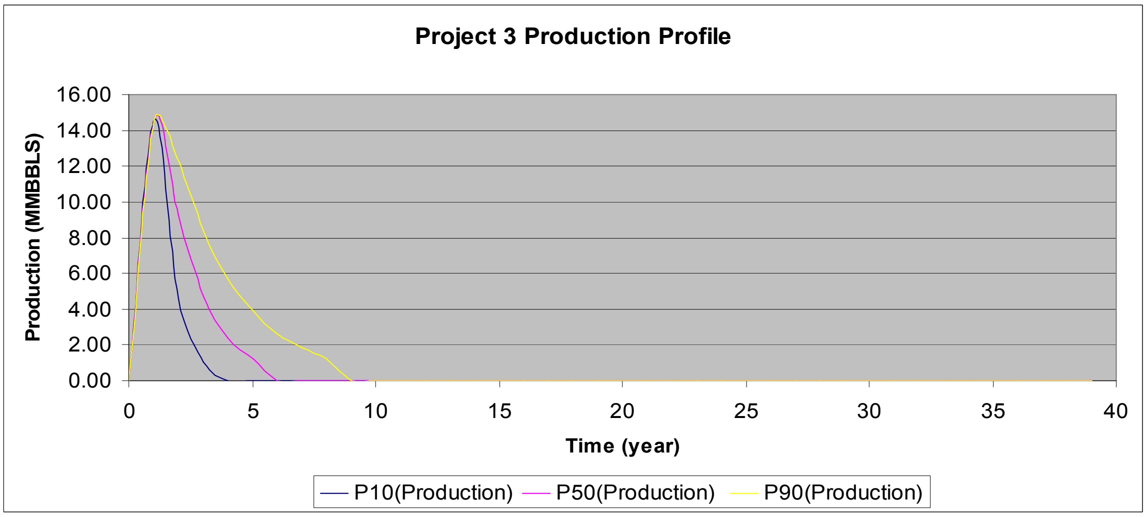


Figure 11-9 P10, P50 and P90 production of project 3

After tax cash flow profile:

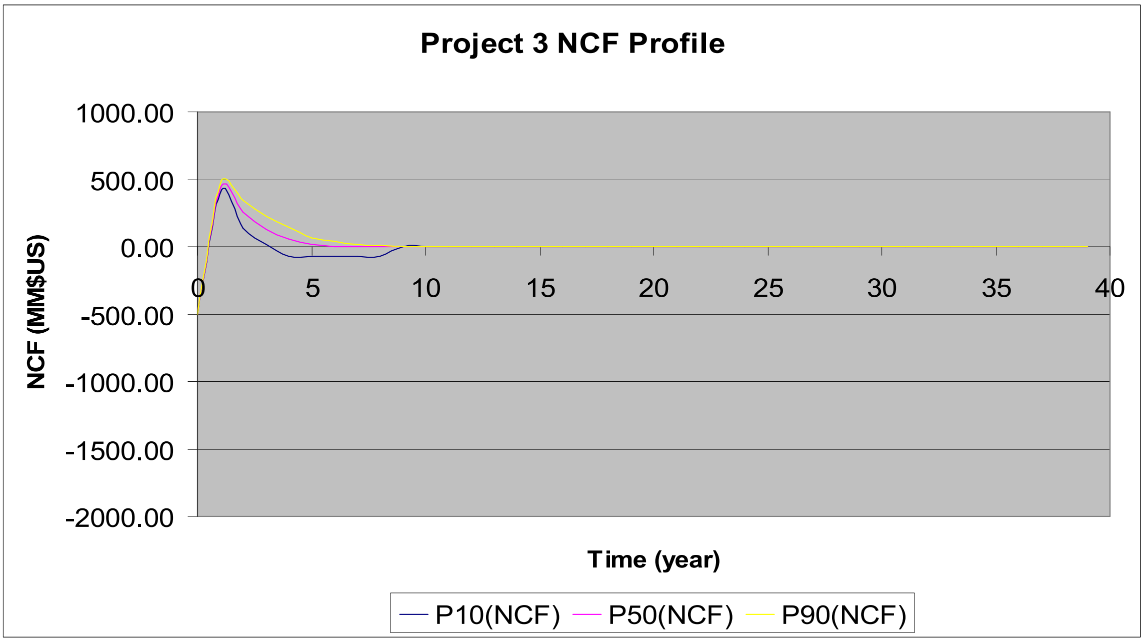


Figure 11-10 P10, P50 and P90 NCF of project 3

NPV distribution:

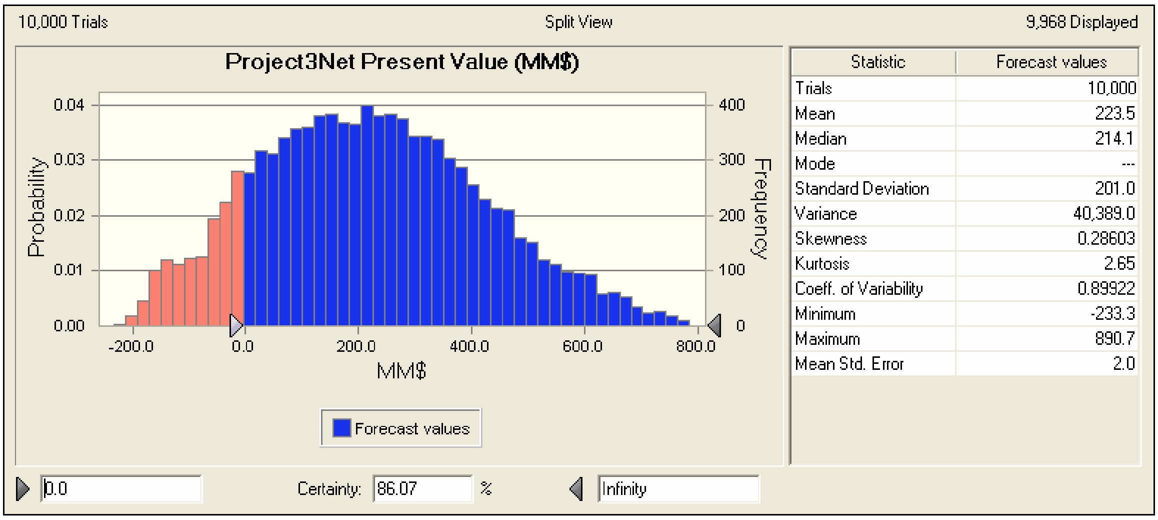


Figure 11-11 NPV distribution of project 3

Reserves distribution:

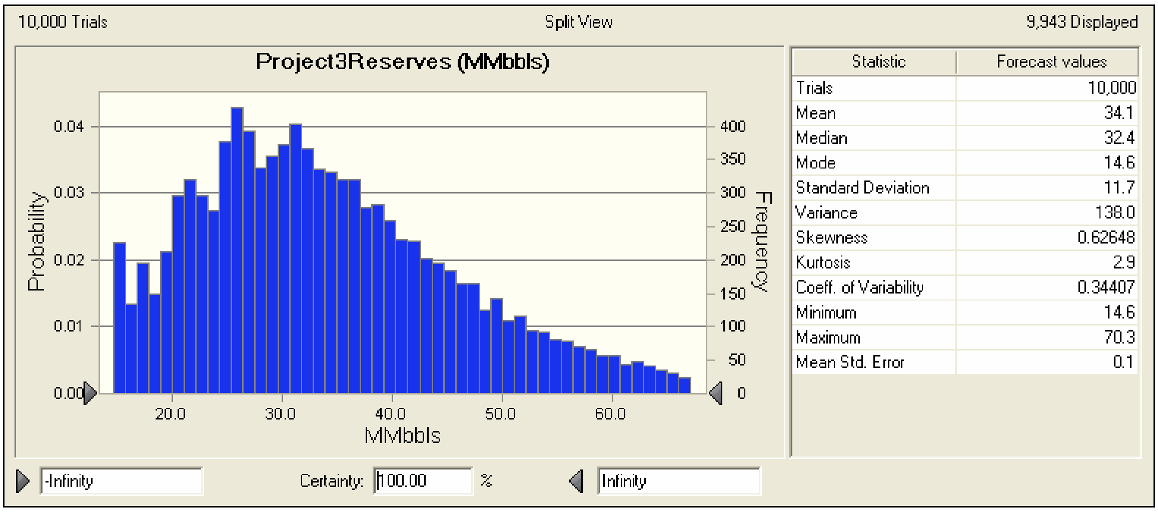


Figure 11-12 Reserves distribution of project 3

11.4 Project 4

Production profile:

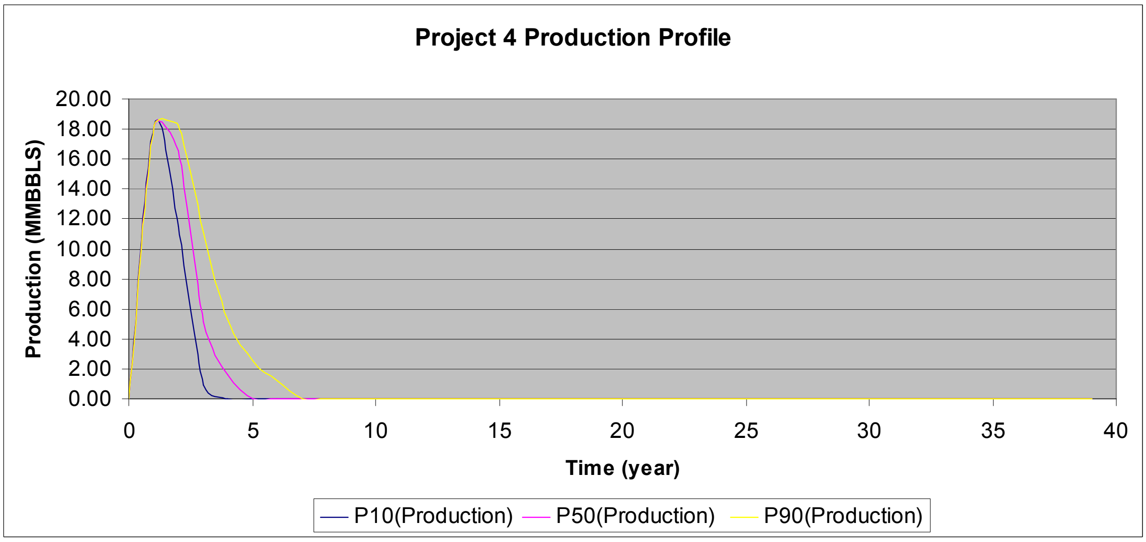


Figure 11-13 P10, P50 and P90 production profile of project 4

After tax cash flow profile:

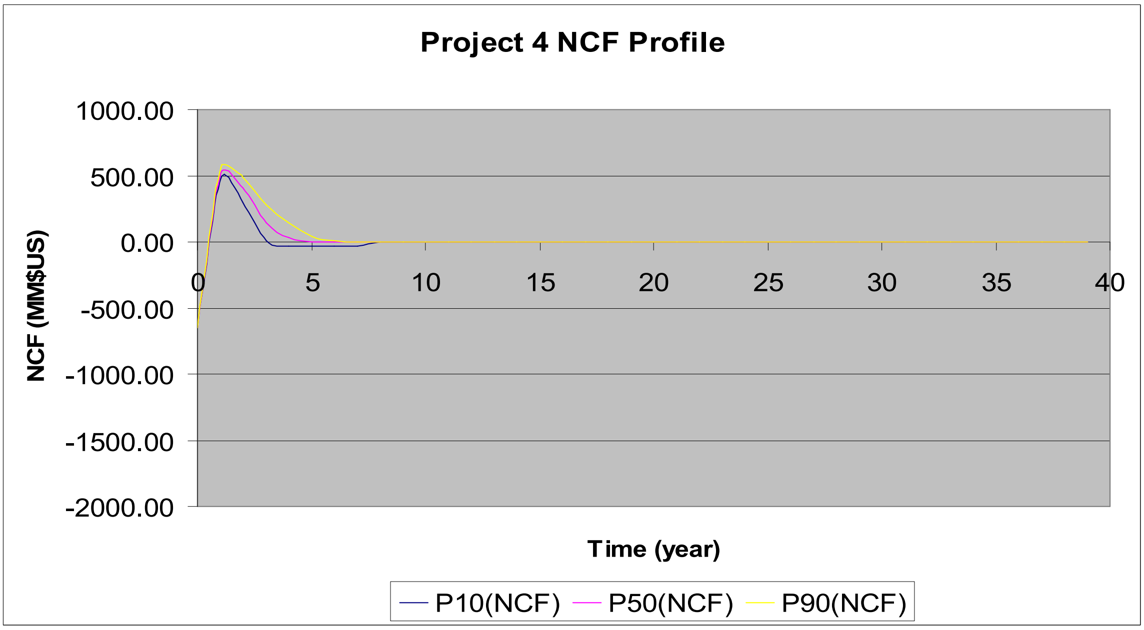
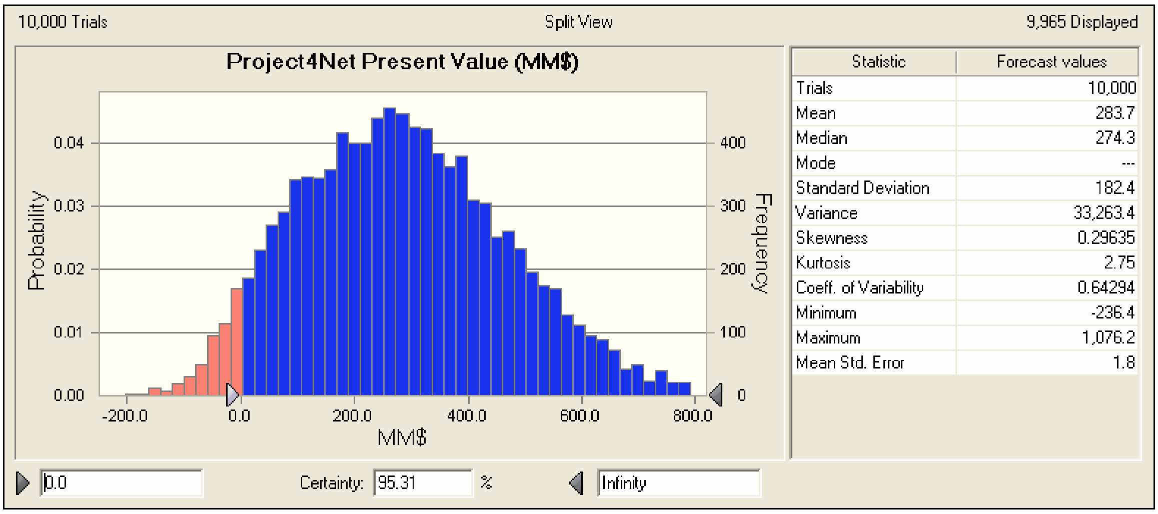


Figure 11-14 P10, P50 and P90 NCF of project 4

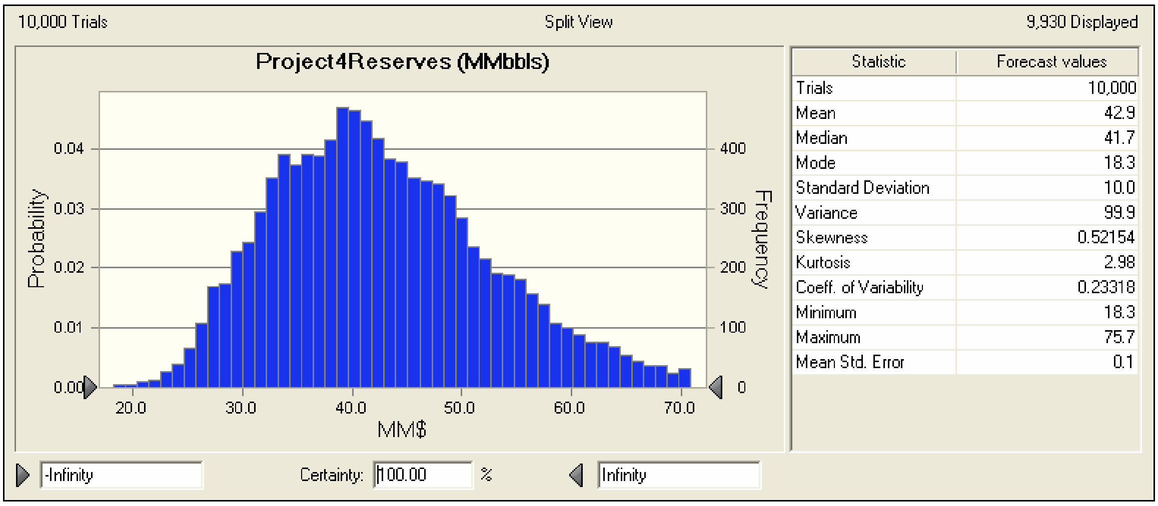
NPV distribution:



Statistic	Forecast values
Trials	10,000
Mean	283.7
Median	274.3
Mode	---
Standard Deviation	182.4
Variance	33,263.4
Skewness	0.29635
Kurtosis	2.75
Coeff. of Variability	0.64294
Minimum	-236.4
Maximum	1,076.2
Mean Std. Error	1.8

Figure 11-15 NPV distribution of project 4

Reserves distribution:



Statistic	Forecast values
Trials	10,000
Mean	42.9
Median	41.7
Mode	18.3
Standard Deviation	10.0
Variance	99.9
Skewness	0.52154
Kurtosis	2.98
Coeff. of Variability	0.23318
Minimum	18.3
Maximum	75.7
Mean Std. Error	0.1

Figure 11-16 Reserves distribution of project 4

11.5 Project 5

Production profile:

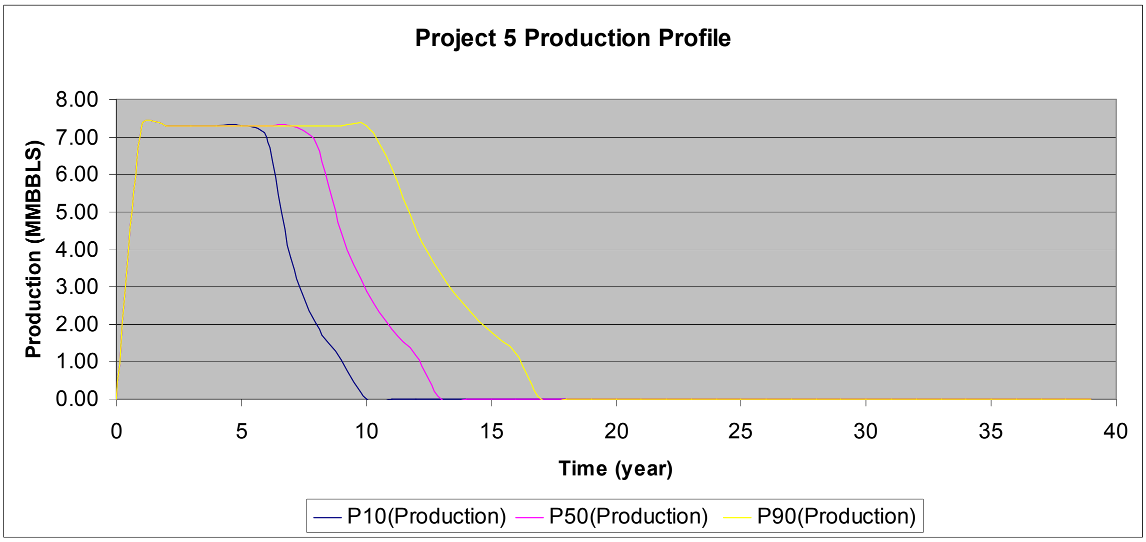


Figure 11-17 P10, P50 and P90 production of project 5

After tax cash flow profile:

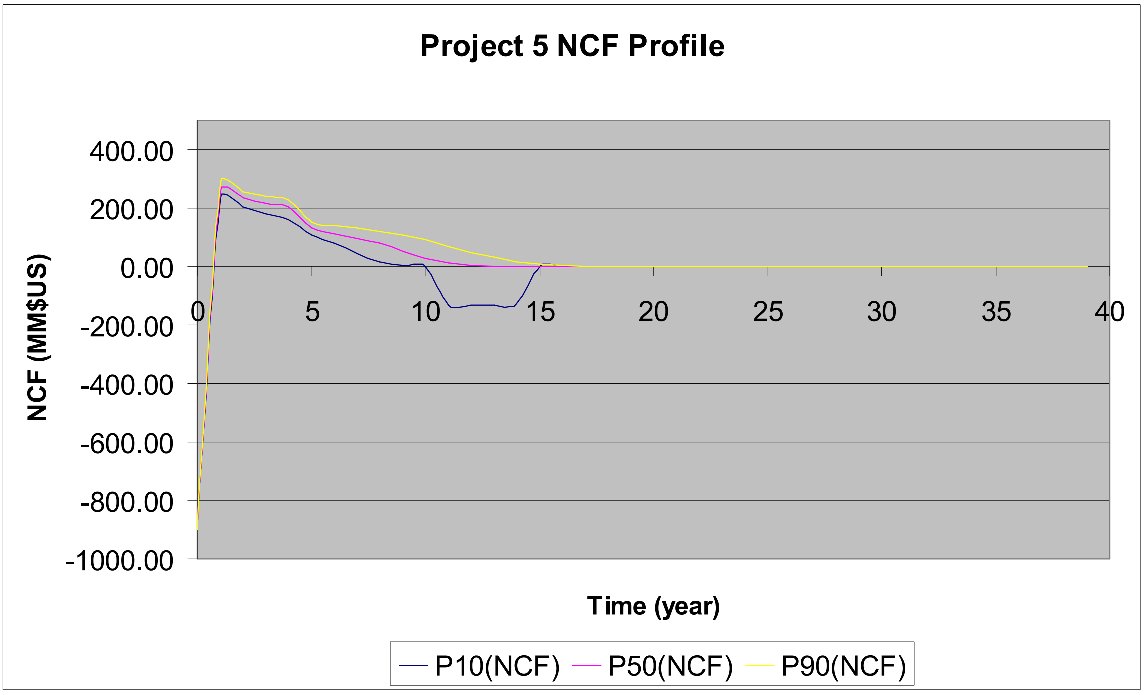


Figure 11-18 P10, P50 and P90 NCF of project 5

NPV distribution:

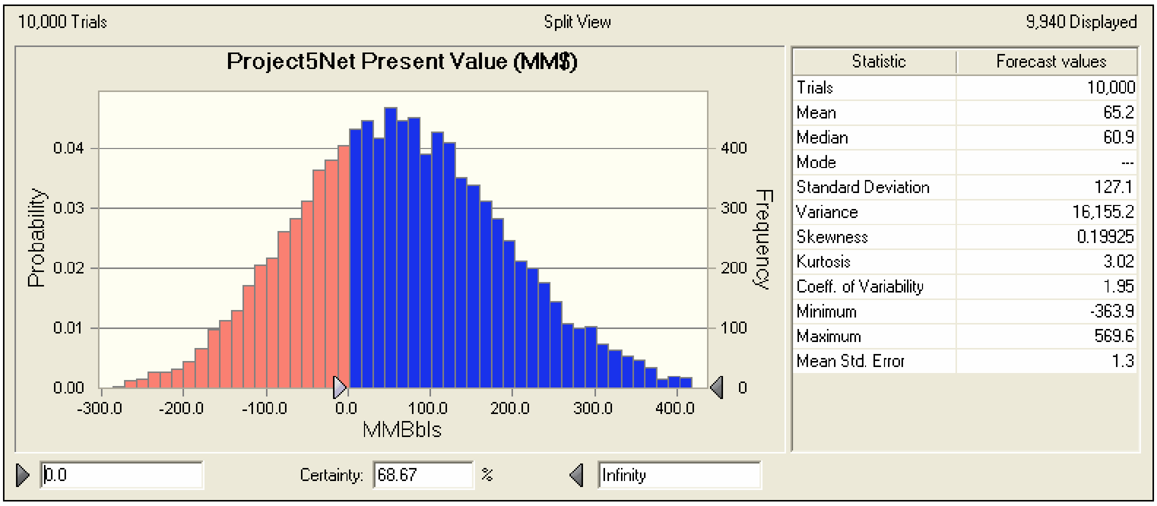


Figure 11-19 NPV distribution of project 5

Reserves distribution:

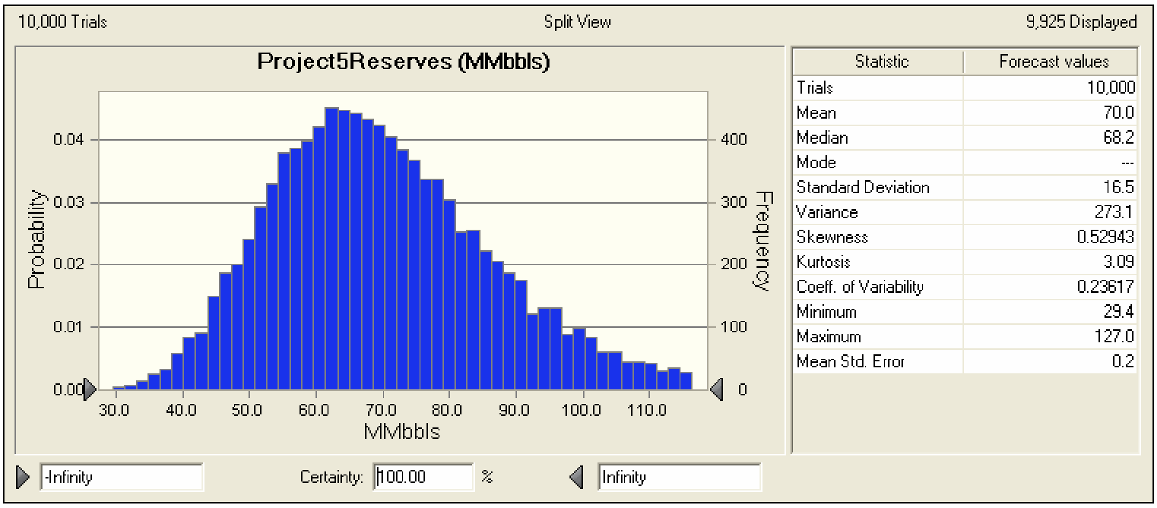


Figure 11-20 Reserves distribution of project 5

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